Facilitator's Manual

JOYFUL TEACHING AND LEARNING OF MATHEMATICS

Central Board of Secondary Education

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Central Board of Secondary Education
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भारत का संविधान

उद्देशिका

हम, भारत के लोग, भारत को एक [1][सम्पूर्ण प्रभुत्व-संपन्न समाजवादी पंथनिरपेक्ष लोकतंत्रात्मक गणराज्य] बनाने के लिए, तथा उसके समस्त नागरिकों को:

सामाजिक, आर्थिक और राजनैतिक न्याय,
विचार, अभिव्यक्ति, विश्वास, धर्म
और उपासना की स्वतंत्रता,
प्रतिष्ठा और अवसर की समता
प्राप्त कराने के लिए
तथा उन सब में व्यक्ति की गरिमा
और [2][राष्ट्र की एकता और अखंडता]
सुनिश्चित करने वाली बंधुता बढ़ाने के लिए

दृढ़संकल्प होकर अपनी इस संविधान सभा में आज तारीख 26 नवम्बर, 1949 ई० (मिति मार्गशीर्ष शुक्ल सप्तमी, संवत् दो हजार छह विक्रमी) को एतद्द्वारा इस संविधान को अंगीकृत, अधिनियमित और आत्मार्पित करते हैं।

[1] संविधान (बयालीसवां संशोधन) अधिनियम, 1976 की धारा 2 द्वारा (3.1.1977 से) "प्रभुत्व-संपन्न लोकतंत्रात्मक गणराज्य" के स्थान पर प्रतिस्थापित।

[2] संविधान (बयालीसवां संशोधन) अधिनियम, 1976 की धारा 2 द्वारा (3.1.1977 से) "राष्ट्र की एकता" के स्थान पर प्रतिस्थापित।

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[1][भाग 4 क

मूल कर्त्तव्य

51 क. भारत के प्रत्येक नागरिक का यह कर्त्तव्य होगा कि वह –

(क) संविधान का पालन करे और उसके आदर्शों, संस्थाओं, राष्ट्रध्वज और राष्ट्रगान का आदर करे;

(ख) स्वतंत्रता के लिए हमारे राष्ट्रीय आंदोलन को प्रेरित करने वाले उच्च आदर्शों को हृदय में संजोए रखे और उनका पालन करे;

(ग) भारत की प्रभुता, एकता और अखंडता की रक्षा करे और उसे अक्षुण्ण रखे;

(घ) देश की रक्षा करे और आह्वान किए जाने पर राष्ट्र की सेवा करे;

(ङ) भारत के सभी लोगों में समरसता और समान भ्रातृत्व की भावना का निर्माण करे जो धर्म, भाषा और प्रदेश या वर्ग पर आधारित सभी भेदभाव से परे हों, ऐसी प्रथाओं का त्याग करे जो स्त्रियों के सम्मान के विरुद्ध हैं;

(च) हमारी सामाजिक संस्कृति की गौरवशाली परंपरा का महत्त्व समझे और उसका परिरक्षण करे;

(छ) प्राकृतिक पर्यावरण की जिसके अंतर्गत वन, झील, नदी और वन्य जीव हैं, रक्षा करे और उसका संवर्धन करे तथा प्राणिमात्र के प्रति दयाभाव रखे;

(ज) वैज्ञानिक दृष्टिकोण, मानववाद और ज्ञानार्जन तथा सुधार की भावना का विकास करे;

(झ) सार्वजनिक संपत्ति को सुरक्षित रखे और हिंसा से दूर रहे;

(ञ) व्यक्तिगत और सामूहिक गतिविधियों के सभी क्षेत्रों में उत्कर्ष की ओर बढ़ने का सतत प्रयास करे जिससे राष्ट्र निरंतर बढ़ते हुए प्रयत्न और उपलब्धि की नई उंचाइयों को छू ले;

[1][(ट) यदि माता-पिता या संरक्षक है, छह वर्ष से चौदह वर्ष तक की आयु वाले अपने, यथास्थिति, बालक या प्रतिपाल्य के लिये शिक्षा के अवसर प्रदान करे।]

[1]संविधान (बयालीसवां संशोधन) अधिनियम, 1976 की धारा 11 द्वारा (3-1-1977 से) अंत:स्थापित।

[2]संविधान (छियासीवां संशोधन) अधिनियम, 2002 की धारा 4 द्वारा (1-4-2010 से) अंत:स्थापित।

THE CONSTITUTION OF INDIA

PREAMBLE

WE, THE PEOPLE OF INDIA, having solemnly resolved to constitute India into a [1]**[SOVEREIGN SOCIALIST SECULAR DEMOCRATIC REPUBLIC]** and to secure to all its citizens :

JUSTICE, social, economic and political;

LIBERTY of thought, expression, belief, faith and worship;

EQUALITY of status and of opportunity; and to promote among them all

FRATERNITY assuring the dignity of the individual and the[2] [unity and integrity of the Nation];

IN OUR CONSTITUENT ASSEMBLY this twenty-sixth day of November, 1949, do **HEREBY ADOPT, ENACT AND GIVE TO OURSELVES THIS CONSTITUTION.**

[1] Subs, by the Constitution (Forty-Second Amendment) Act. 1976, sec. 2, for "SOVEREIGN DEMOCRATIC REPUBLIC" w.e.f. 3.1.1977)
[2] Subs, by the Constitution (Forty-Second Amendment) Act. 1976, sec. 2, for "unity of the Nation" (w.e.f. 3.1.1977)

THE CONSTITUTION OF INDIA

[1][PART IV A

FUNDAMENTAL DUTIES

51A. Fundamental Duties- It shall be the duty of every citizen of India-

(a) to abide by the Constitution and respect its ideals and institutions, the National Flag and the National Anthem;

(b) to cherish and follow the noble ideals which inspired our national struggle for freedom;

(c) to uphold and protect the sovereignty, unity and integrity of India;

(d) to defend the country and render national service when called upon to do so;

(e) to promote harmony and the spirit of common brotherhood amongst all the people of India transcending religious, linguistic and regional or sectional diversities; to renounce practices derogatory to the dignity of women;

(f) to value and preserve the rich heritage of our composite culture;

(g) to protect and improve the natural environment including forests, lakes, rivers rivers, and wild life, and to compassion for living creatures;

(h) to develop the scientific temper, humanism and the spirit of inquiry and reform;

(i) to safeguard public property and to abjure violence;

(j) to strive towards excellence in all spheres of individual and collective activity so that the nation constantly rises to higher levels of endeavour and achievement;

[2][(k) who is a parent or guardian to provide opportunities for education to his/her child or, as the case may be, ward between age of six and forteen years.]

[1] Ins. by the Constitution (Forty-Second Amendment) Act, 1976, s.11 (w.e.f. 3-1-1977).
[2] Ins. by the Constitution (Eighty-Sixth Amendment) Act, 2002, s.4 (w.e.f. 1-4-2010).

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INTRODUCTION

The National Policy on Education 1986 proposed that "Mathematics should be visualized as the vehicle to train a child, to think, reason, analyze and to articulate logically. There is a shift in treating mathematics education as an instrument for the National development to the development of child's abilities. This has been carried forward further in the National Curriculum Framework 2005 wherein child's active engagement in mathematics learning involving "inquiry, exploration, questioning, debates, application and reflection leading to theory building and creation of ideas/positions in Mathematics" is envisaged .

Objectives of teaching Mathematics:

The objective of school mathematics curriculum at elementary and secondary level is to provide students opportunities to be equipped with important Mathematics needed for better educational, professional and social choices. It empowers students to investigate, to make sense of and to construct mathematical meanings from new situations. By learning Mathematics, students must be able to develop logical thinking and reasoning. They must also be able to find out Mathematics in all walks of life including art. They should not find the subject in isolation with other subjects or life, rather be able to make use of their learning in day to day life.

Curricular Expectations:

According to the Learning outcomes document prepared by the NCERT, the curricular expectations from Mathematics in **upper primary classes (VI- VIII)** are as under :

Children are expected to:

a) move from concrete ideas of numbers to number sense

b) see relationships between numbers and look for patterns in relationship

c) understand and apply concepts related to variables, expressions, equations, identities, etc.

d) use arithmetic and algebra to solve real life problems and pose meaningful problems

e) develop aesthetic sense by discovering symmetries in shapes like triangles, circles and quadrilaterals

f) identifies space as region enclosed within boundaries of a shape

g) develop spatial understandings in terms of perimeter, area and volume and use them to solve day to-day life problems

h) learn to provide reasoning and convincing arguments to justify her/his own conclusions in mathematical context

i) collect, represent (graphically and in tables) and interpret data/information from her/his life experiences.

Present status of Mathematics Learning

Various educational surveys and achievement data over the years show that learning achievements of children in various subjects-especially Mathematics- are not up to the expected levels despite all the efforts made by states in this direction. It is a fact that many a time teachers complete the syllabus as per the textbook but they do not have a clear idea about the kind of learning they expect from children in various subjects including Mathematics. The curricular expectations as defined by NCERT are not met.

Importance of teacher empowerment

Learning is a continuous process. The learning outcomes are impacted by the learning and pedagogical processes used to develop competencies. The learners are expected to realise and use Mathematics as an important tool that they can talk about, use and explore as well as understand its structure. This will happen only if the teachers have necessary competencies and motivation to do so. Thus, teacher empowerment plays a crucial role in this respect.

About this Manual

Many students in our schools even today show displeasure towards the learning of Mathematics. Students view Mathematics as dull, boring, and stereotyped. There are not enough instances when a teacher has tried to teach Mathematics in a different way, say through activities and games. These activities and games make the process of learning more interesting and effective. The manual provides inputs for teachers to make the teaching and learning joyful by engaging students in meaningful activities.

The Manual is developed by keeping in mind the following aspects:

1. Importance be given to building teacher competencies in contextualization of Mathematics with references from daily life for the purpose of promoting joyful learning
2. Sessions are to be interactive and participatory in nature
3. Main focus areas should be content, pedagogy, assessment and educational tools & technology. Tips are also to be given on creating conducive environment in and around the school.

Schedule of the Programme

DAY-1

Time	Session
8:30 am - 9:00 am	Registration and other formalities
9:00 am – 10:00 am (1 hour)	**Session 1**: Ice Breaking and Introduction
10:00 am - 10:45 am (45 minutes)	**Session 2:** Innovations in Mathematics Teaching: From Idea to Practice
10:45 am – 11:00 am (15 minutes)	**Break**
11-00 a.m -11-45 a.m (45 minutes)	**Session 3:** Innovative teaching of Mathematics
11:45 am - 01:00 pm (1: 15 minutes)	**Session 3:** Innovative teaching of Mathematics- Ideas of Participants
1:00 pm -2:00 pm (1 hour)	**Lunch Break**
2:00 pm – 03 : 15pm (1:15 minutes)	**Session 5:** Recreational Mathematics
3.15 pm – 3.30 pm	**Break**
3:30 pm – 4:30 pm (1 hour)	**Session 6:** Learning from History of Mathematics

DAY-2

Time	Session
9:00 am - 10:00 am (1 hour)	**Session 1:** Handling common errors
10:00 am - 11:00 am (1 hour)	**Session 2:** Handling difficult concepts
11:00 am - 11:15 a.m (15 minutes)	**Break**
11:15 am - 01:00 pm (1 hour)	**Session 3 :** Using IT in teaching Mathematics
1:00 pm - 2:00 pm (1 hour)	**Lunch Break**
2:00 pm - 03: 00pm (1 hour)	**Session 4:** Using IT in teaching Mathematics (Hands on)
3.00 pm - 3.15 pm	**Break**
3:15 pm - 4:00 pm (45 minutes)	**Session 5:** Joyful Assessment
4:00 pm - 4:30pm (30 minutes)	**Session 6:** Creating Culture for Mathematics Learning
4:30 pm - 5:00pm (30 minutes)	Feedback and Valedictory

Note for Facilitator

Mathematics is often considered a dreaded subject for learners. This fear of Mathematics makes it difficult for teachers to make Mathematics learning enjoyable for learners. Teachers have to make learning Mathematics an interesting task for learners despite the constraints of time and resources. Addressing individual differences in learners is also a challenging task for teachers. Apart from planning and enhancing one's knowledge and skills, this requires a will to improve the teaching-learning process. This manual is an attempt to facilitate teachers in this endeavour.

As a facilitator , you are expected to provide course participants with a situation to bring out the knowledge and skills of participants who are experienced, creative and resourceful. A facilitator has to use his or her experience and expertise in mentoring the course participants and encourage the art of self-analysis and reflection. This manual provides some guidelines in this regard but in no way does it limit the scope of discussions or the sessions. As per the interest, experience and proficiency of the course participants, you are advised to make suitable additions and modifications in the content being discussed. Further, it is imperative that the facilitator thoroughly reads this manual and makes arrangements for any additional resources required during this programme.

Things to check before the workshop

- **The teaching rooms:** Adequate furniture/seating, board, chalk, ventilation, fans, notice board, electric points etc.
- **Break times:** Arrangements for tea, meals etc.
- **Transport arrangements** (wherever applicable)
- **Resources:** Location and availability of reprographic facilities, paper, pens, markers, speakers for audio and video clips and other material.
- **Workshop materials:** Adequate supplies of all the handouts, CDs with PPTs and resource videos with internet connection, etc., writing pads for teachers, folders, pens, etc.
- **Installation of Geogebra:** It may be ensured that Geogebra classic 5 Software is installed in all computers at the venue well in advance.

Things to take in your Kit bag

- NCERT Mathematics text-books for Class VIII to X (The course participants to be instructed beforehand to bring the NCERT Mathematics textbooks & Geometry boxes for the programme)
- Copies of all the handouts and workshop materials
- In addition to the stationery available at the venue, ensure availability of: Cello-tape, chalk & duster, chart papers, markers, scissors, glue sticks, sketch pens, origami sheets and graph papers.

If you are working in a team of two facilitators

- Before the start of the programme, divide work equally for the next day, and write down what you agreed upon.
- Set aside a short time for review and planning each day; make it clear to participants that you need that time and that it is sacrosanct.
- Play to your individual strengths as (e.g., one may be better at board writing, one may be better at talking to the whole group).
- Never disagree in public.
- Don't compete with each other or interrupt each other: if you are seen to listen carefully to each other and to respect each other, this will have a positive effect on the course participants.
- Give each other constructive feedback after sessions (this is a development opportunity for you); positive comments are as valuable as negative ones. Give feedback to each other on matters such as timing, voice, eye-contact, blackboard work etc. Remember to appreciate the good work of your co facilitator from time to time.
- When teachers are working in pairs or small groups, decide amicably who is to monitor which half of the group.
- Develop the habit of working briskly, and adhering to the time allocations for each activity.
- Remember that you can run parallel sessions for some activities (2x30 may be appropriate and easier to handle than 1x60).

Some Do's

- Arrive early for the first session, arrange the seating to your liking, and make sure you are there to welcome participants in a warm and encouraging way.
- Encourage active participation.
- Listen, actively and openly, to participants' views.
- Ensure that course participants (CPs) listen to each other in discussion.
- Acknowledge and affirm participants' contributions.
- Clarify their doubts.
- Know your plan for each session.
- Agree on a signal (a raised hand) to stop discussions without stress and strain.
- Ensure that each individual has an equal chance to contribute.
- Repeat instructions before starting a session or an activity.
- Maintain eye contact with participants during plenary sessions.
- Prepare posters and/or b/w board with important messages to save time (e.g. aims of a particular session).
- Remember to write page numbers on the b/w board, in addition to orally telling teachers to find a particular page in the handout.
- Start in time each morning, and stick to the agreed breaks. You can easily lose 10-15% of precious time if you don't keep an eye on your watch.
- Make sure that you have prepared and checked the handouts, videos and audios.

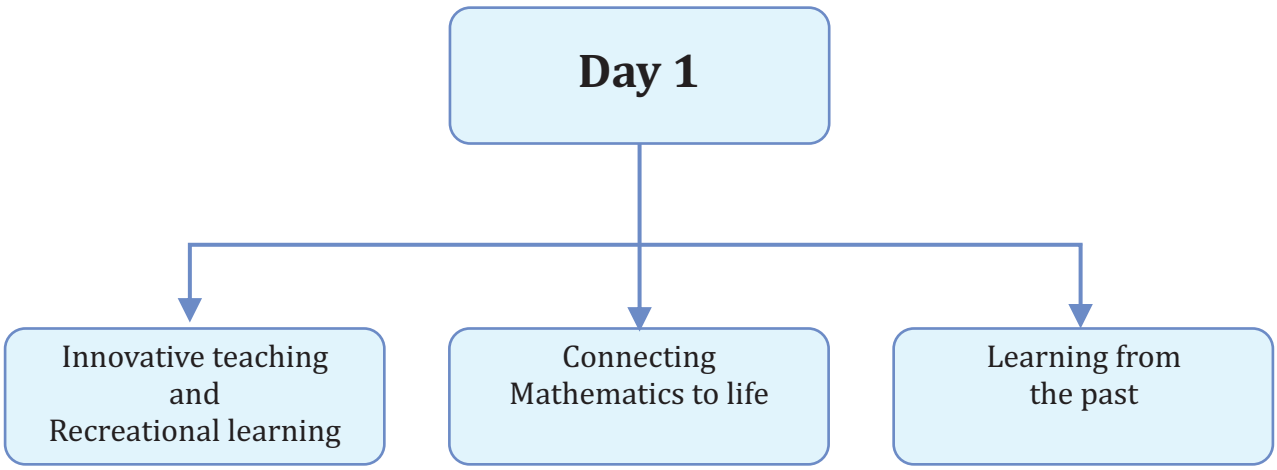
Some Don'ts

- Don't try to force your views; it is better to admit that you don't know everything.
- Don't put down any participant or their contribution.
- Don't be sarcastic.
- Don't treat participants like students.
- Don't allow yourself to become defensive about the material.
- Don't criticize the materials, education system, policy and infrastructure publicly.
- Don't talk too much.

- Don't allow any participant to hijack a discussion.
- Don't take any criticism personally.
- Don't be too "driven" by the materials and remain open to positive changes.
- Don't allow local administrative demands and procedures to consume the time you need for the workshop.
- Don't interrupt pair/small group work once you have set it up–except to announce "two more minutes".

Advice on dealing with difficult questions

- With your fellow facilitator, note down any objections or questions which you think might be raised and discuss strategies for dealing with them.
- Play for time when confronted with a difficult question (e.g.by asking for the question to be rephrased, or by inviting participants' view).
- Don't get involved in disputes with individual participants.
- Don't allow participants to shout at each other.
- If there is a disruptive participant, make sure you find a busy role for him/her, especially in group work (e.g. where he/she could function as a secretary); then give positive feedback whenever you have an opportunity.
- Remember you are running the workshop; you should resist, quietly but firmly, any attempt to usurp your role.
- Have a quiet word with any awkward participant who doesn't calm down within a day. Find out what is bothering them and discuss it.
- Remember that this is only a first step in the long process of change for the participants. You should expect awkward questions and resistance. You can help as a facilitator by allowing participants to clear their doubts, by being patient and supportive, and by making provision for maximum opportunities to experience alternative methodologies and to consider their implications. Responsibility for change is theirs, not yours.
- Be prepared for negative feedback too. You may not see the fruits of your labour during the workshop itself. Participants will need time to reflect after the workshop and to try out the activities in their own classrooms.
- A box with 'Parking Lot' written may be kept for questions which are to be taken at a later stage.


Day 1
Innovative teaching and Recreational learning
Connecting Mathematics to life
Learning from the past

SESSION 1

Ice Breaking and Introduction

(Duration : 1 hour)

Objectives:

At the end of this activity, the participants will be able to:

- know each other and the facilitator
- compare the objectives of the programme with their expectations
- appreciate their role in making teaching and learning Mathematics joyful

Preparation :

Power point presentation

Procedure :

1. Make adequate number of groups by asking each participants to tell name of one of the geometrical figures (say Triangle, Square, Rectangle, Circle, rhombus, trapezium, Cone, Cylinder, cube etc.) one after the other. Group all the people with same name of the figures.

   Or

   Ask the participants add the digits of their first name with the help of the following table. Participants with same number will be in one group. Facilitator is free to mix or divide groups as per requirement.

1	2	3	4	5	6	7	8
A	B	C	D	E	U	O	F
I	K	G	M	H	V	Z	P
J	R	L	T	N	W		
Q		S		X			
Y							

2. Show any picture on Power Point and ask the group to describe the picture mathematically or construct a mathematical problem based on this picture and ask each participant to introduce himself/herself in Mathematical language.

3. Facilitator explains the importance of connecting life to Mathematics then shares one video showing this connection : https://www.youtube.com/watch?v=dpv06SFHtRg

4. Ask the participants about the expectations from the workshop.

5. The facilitator writes down the expectations on the board and explains the objectives of the workshop.

Objectives of the Programme:

After the completion of the sessions, participants will be able to

- appreciate the beauty of Mathematics
- connect Mathematics to day to day life
- make use of innovative teaching strategies.
- develop assessment for joyful learning
- use technology to enhance learning

Sample problem using the picture:

If number of drops taken from one of the eight sections of the orange is 100 with volume of each drop is 0.1 ml then find out the radius of the orange assuming it to be a sphere.

Facilitator can also ask the participants to connect different concepts to life. e.g. What is the real-life application of the idea that sum of interior angles is 180 degrees in any triangle.

SESSION 2

Innovative teaching in Mathematics

(Duration : 45 minutes)

Objectives:

At the end of this session, the participants will be able to:

- appreciate the scope and possibilities of innovations in Mathematics classroom
- introduce innovations in classroom practice
- connect teaching to life around

Preparation: Materials as required by the participants

Procedure:

1. **The session will start with introspective questions:**
   - ➢ Are your students afraid of Mathematics?
   - ➢ Do majority of your students not enjoy Mathematics learning?
   - ➢ Do your students find Mathematics boring and difficult?
   - ➢ Are your students conceptually weak in Mathematics?
   - ➢ Do most of your students score poor marks in Mathematics?

If answer to majority of these questions is yes, then, it is the time for a teacher to bring innovation in the pedagogy.

2. The facilitator explains innovation with examples given in the box below.
3. The facilitator explains the connectivity of variable with daily life by giving example of Nehru Place ,Delhi and asks each group to make a list of all the 'players' or 'elements' in the setting as variables or constants as per handout . Some examples are: the numbers of car parked in the area, the hawkers or the number of shops on the first floor.
4. Facilitator asks the participants to identify how the variables and constants are related to each other and bring out the following response from the particpants with regards to expression in Mathematical form .
   - ➢ First decide which variable(s) will you focus on.
   - ➢ Consider a logical relationship of that variable with different quantities.

- Now decide which quantifiers we would use in the relationship(s) described in point 2 above. Write these as a mathematical expression. For example, we could state that we would need one police officer for a combination of every ten shops. Therefore number of police officials can be described as 0.1s (where 's' is the number of shops)
- We can predict the range of values for each variable. In cases where we are having difficulty predicting a range, identify the reasons for the difficulty. For example, the number of escalators cannot be less than one, because we cannot have half of an escalator. We also cannot have an unlimited number of escalators, because they take up space. Deciding on the maximum number of escalators is harder to do because it will depend on several factors.
- Which of the variables do we think can be controlled easily? Controlling a variable could mean that we either can restrict its range or fix its value without affecting the situation very much.
- Now consider the above points and try making different expressions taking into account various dynamics of Nehru Place.

5. Facilitator concludes the session by consolidating the importance of innovations in Mathematics teaching and discussing the ways of mathematisation of real life problems.

Understanding innovation

A) Innovation is an idea:

that is novel and useful;

that has economic or social value;

that can be sustained and scaled up;

that addresses a problem to bring intelligent solution.

B) Innovation in Education

Innovation in education is any reform, practice, strategy or a resource that can make learning a joyful and long lasting experience for the learner.

C) Innovation in Pedagogy

shall make learning experiences more meaningful and productive for learners;

shall make learning opportunities more accessible;

shall make impact of teaching stress free and long-lasting.

Any idea, how innovative it may be, if not implemented properly can be a disaster. This is more crucial in teaching because we deal directly with children and their learning.

Idea + Implementation + Review/Monitoring = Design of Innovation

Any innovation in Mathematics teaching and learning shall help in:

simplifying the description, visualization or application of a concept;

diversify the concept and its application;

presenting the concepts in different forms;

abstraction and generalization of concepts;

breaking the monotony in learning.

We can bring innovation in Mathematics teaching by using technology and using alternate strategies such as dialogue, discussion, problem solving, learning by doing, questioning, story boards etc.

Below is an example of innovation in a Mathematics classroom:

Example: Algebra (Grade VI-VII)

(Description: This is an example of a real time situation. This context will be used to help students choose variables, form equations, represent equations, solve equations and infer the results. It helps to see application of Algebra in real life and strengthen the fundamental concepts in Algebra).

Context:

Nehru Place in Delhi, Asia's largest market for computers and peripherals, is always crowded. The area is always full of hustle and bustle.

We can develop models to predict and describe such real life situations. This mathematical modeling relies on deciding what the variables are (the numerical quantities that will vary) and what the constants are (the quantities that will stay the same) in this setting.

Participants are marked to find out the constants & variable and mention in the handout given.

Handout - 1

Identify variables and constants

S. No.	Variables	Constant	Reason

Additional inputs to participants:

Description: This is an example on how to promote mathematical thinking and to hold discussion in the class. It will allow students to make a choice and defend their choice. It will help teachers to unearth the misconceptions developed by students.

Task

Make your own group

Here is a list of terms associated with Statistics:

frequency, deviation, data, central tendency, observations, histogram, arithmetic mean, median, mode, frequency distribution, pie chart, bar graph, average, double bar graph, range, population, sample, represent, infer class, interval, raw data, histogram, pictograph, tally marks, upper limit of class interval, class size.

Organize them in different groups under following conditions:

There must be at least three groups.

Each group must contain at least three terms.

Terms can be placed in more than one group.

Group 1	**Group 2**

Group 3

SESSION 3

Innovative teaching in Mathematics

(Duration : 45 minutes)

Objectives:

At the end of this activity, the participants will be able to:

- appreciate innovative teaching learning in Mathematics
- develop ideas for innovations in teaching

Preparation : Handouts and videos through internet

Procedure:

1. Facilitator shows some innovative ways of teaching by sharing one or two videos given below:

   Concept of equation and variable can be introduced by the activity as given in the video: https://www.youtube.com/watch?v=VgqdsS6cnSY

   Solving a single variable equation : https://www.youtube.com/watch?v=5EQ6_nuE3cI

   Integration of Music in learning linear equation : https://www.youtube.com/watch?v=hK_LQaNDxi4

2. Facilitator explains one innovative activity design using the handout .
3. Inputs are given by the facilitator on how such activities can be conducted.
4. After giving the example, the facilitator asks participants in groups to plan their innovative teaching practices to be shared.

Handout -2

Task oriented problem solving

Time Required: 1 hour

Task: Exploring all possible shapes with same perimeter and different area and vice-versa.

Learning Objectives: Students will be able to :

- develop habit of experimentation.
- learn to record their observations and compare the results
- Internalize that two figures can have the same area, yet different perimeters and also vice versa.

Key Concepts: Area, Perimeter

Material Required: Paper, pen, pencil, graph paper

Warming up questions:

(a) What is perimeter?

(b) What is area?

(c) What is the perimeter & area of a rectangle?

(d) What are the units of perimeter and area, given all the dimensions in cm?

(e) How will you design your study table if your father tells you that its area shall be 36 square units, but it can have any perimeter?

Procedure:

Step 1:

Students will be encouraged to think of possible dimensions of the rectangle whose area is 36 cm^2. All the possibilities will be discussed and drawn.

Step 2:

They will compute perimeter of each of these rectangles. Students will be given graph paper and told to draw all the rectangles having area 36cm^2.

Next find out the perimeters of all the figures. Out of these which one is having least perimeter?

Example: few samples are shown below:

Can you draw any other shape other than rectangle and square having same area?

Draw any 7 shapes and compute their perimeters (1 box = 1 sq. cm).Colour the one with least perimeter.

Example: Two of the samples are shown below:

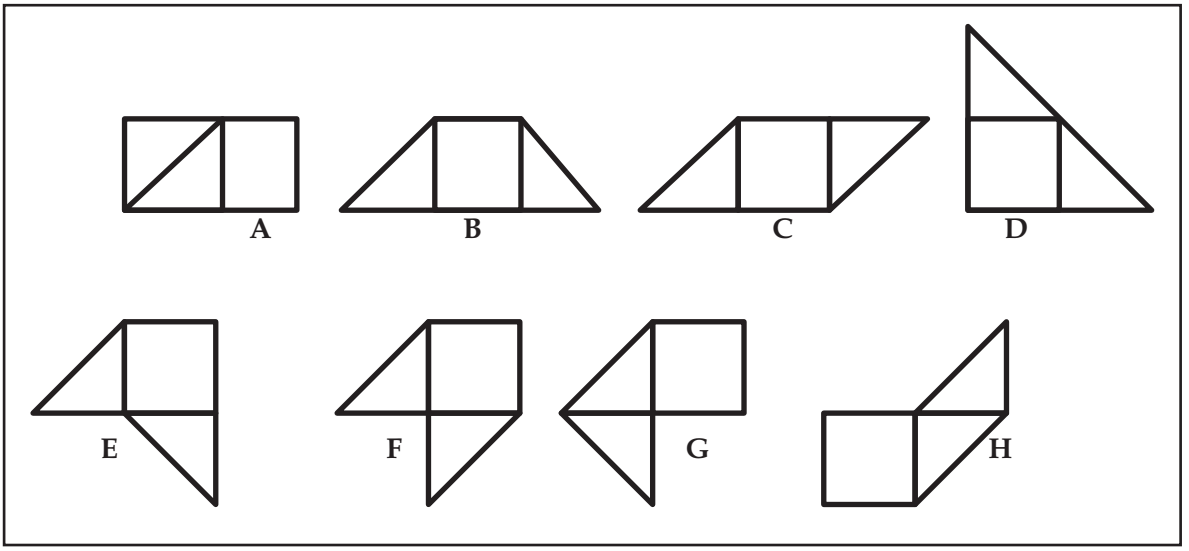
9	By	4												
							6	By	6					
		3	By	12										

Follow up worksheet

1. Draw rectangles with perimeter 24 cm. Also compute the area of each one of them.

   (1 box= 1 sq. cm)

2. Find the area & perimeter of the following:
   (given I box = I sq. cm)

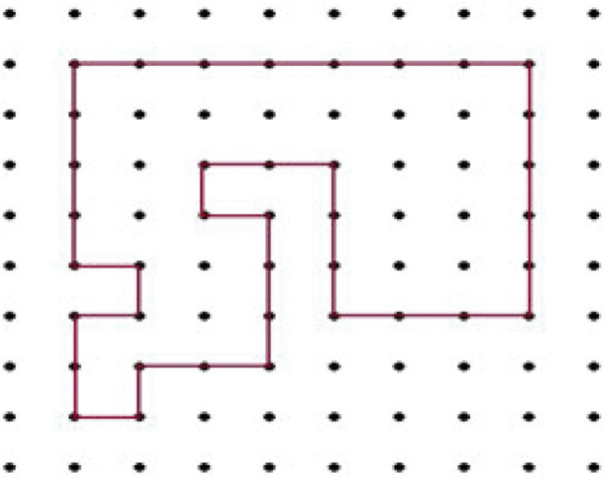


ANS.	Area	Perimeter
(A)	…….	…….....
(B)	…….	……..
(C)	……..	……
(D)	……..	……
(E)	……..	……
(F)	……..	……
(G)	……..	……
(H)	……..	……

2. Find the perimeter & total area covered by the following: (given I box = I sq. cm)

   Ans. Area = ……, Perimeter = ……..

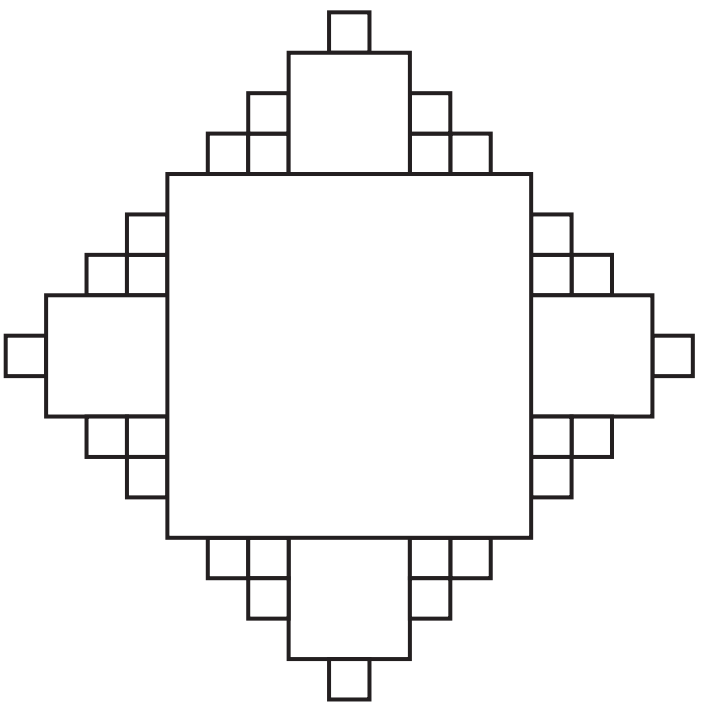
3.



Ans. Area = …… , Perimeter = ……..

4. Find the area & perimeter of the following: (given I box = I sq. cm)

Area = ……



Innovative teaching of Mathematics - Ideas of participants

(Duration : 1:15 minutes)

Objectives:

At the end of this activity, the participants will be able to:

- share the innovative practices adopted in teaching Mathematics in the classrooms
- learn from the peer group

Preparation: Internet connection, Materials as required by the participants

Procedure:

1. Five or more groups are made as per number of participants and each group will present one innovative method of teaching.

2. Other group will observe and give inputs for improvement.

3. Facilitator connects these methods to traditional methods.

Inputs to the Facilitator:

Following are some activities which can also be shared if participants are unable to come out with innovative ideas.

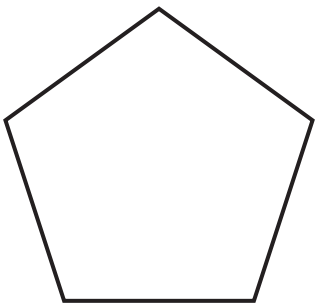
Activity 1

Triangle to polygons

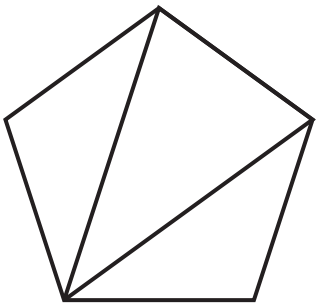
Materials required: Paper, scissors, ruler, pencil.

Procedure:

1. Ask the participants (in groups), to draw various closed polygons of different sides (regular or irregular). e.g.



1. Cut their figures in triangles as shown below:



1. Count the number of triangles formed. In the above case it will be 3. Repeat this for some other figures.
2. Hence the derivation of the formula:

   Sum angle of a Polygon of n sides = (n – 2) X 180^0
   through inductive deductive method

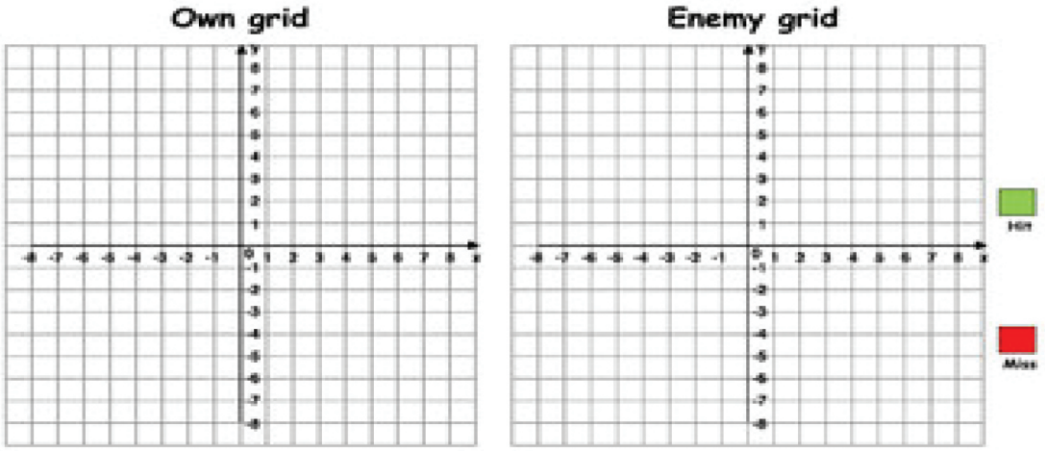
Note : Participants may be asked to connect the activity with the concepts and modify it, if required.

Activity 2
Battleships

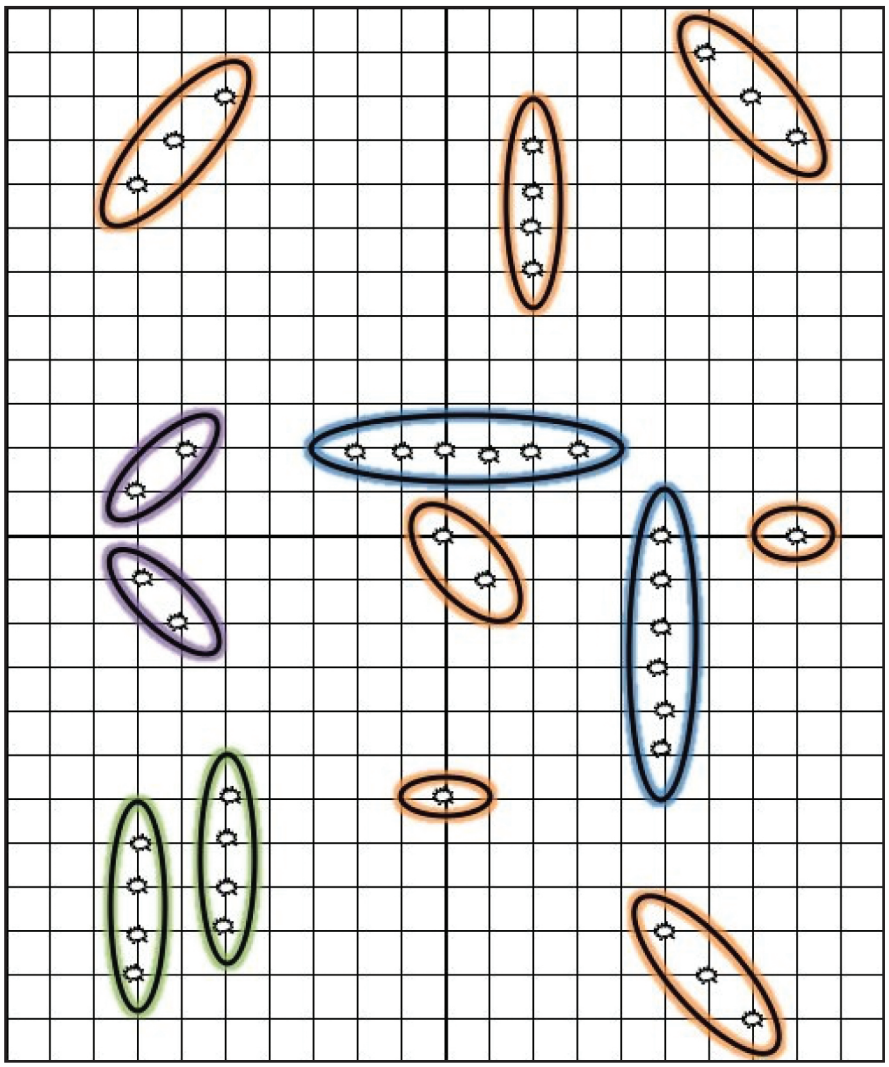
Materials required: Graph papers, coloured pencils, ruler.

Procedure:

1. The game is played on four grids, two for each player.
2. The grids are typically square – usually 10×10 – and the individual squares in the grid are identified by their coordinates.
3. On one grid the player arranges ships and records the shots by the opponent. On the other grid the player records their own shots.



4. Before play begins, each player secretly arranges their ships on their primary grid. Each ship occupies a number of consecutive points on the grid, arranged either horizontally vertically or diagonally. The number of points for each ship is determined by the type of the ship



5. The ships cannot overlap (i.e., only one ship can occupy any given points in the grid). The types and numbers of ships allowed are the same for each player. For example:

Sr. No.	Class of ship	Number	Size
a.	Battleship	One	(4 points)
b.	Destroyer	Two	(3 points)
c.	Submarine	Three	(2 points)

d. After the ships have been positioned, the game proceeds in a series of rounds. In each round, each player takes a turn to announce a target coordinates in the opponent's grid which is to be shot at.

e. The opponent announces whether or not the point is occupied by a ship, and if it is a "miss", the opponent player marks their primary grid with a black pencil; if a "hit" they mark this on their own primary grid with a red pencil.

f. The attacking player notes the hit or miss on their own "tracking" grid with the appropriate colour pencil (red for "hit", black for "miss"), in order to build up a picture of the opponent's fleet.

When all of the squares of a ship have been hit, the ship's owner announce the sinking of either the Submarine, the Destroyer, or the Battleship.

g. If all of a player's ships have been sunk, the game is over and their opponent wins.

Initially the game can be introduced only for the first quadrant, progressing to other quadrants as students gain proficiency.

(Source: www.teachingideas.co.uk)

Note : Participants may be asked to connect the activity with the concepts and modify the activity, if required.

SESSION 5

Recreational Mathematics

(Duration : 1:15 minutes)

Objectives:

At the end of this activity, the participants will be able to:

- share the Mathematics recreational contents like jokes, cartoon, stories, poems etc
- link the recreational Mathematics to the school curriculum

Preparation : Power point presentation

Procedure:

1. Facilitator shows following video and motivate the participants to share the recreational mathematic content. https://www.youtube.com/watch?v=vSNUiQj8ZF4
2. Facilitator gives some other examples of recreational Mathematics.
3. Then the facilitator asks the participants to share jokes, poems, cartoons, ideas , activities etc
4. Facilitator also asks the ways and means of integration of such activities in the classrooms and to the curriculum.
5. Facilitator emphasize the importance of recreation in the classrooms promoting Mathematics learning. He/she also gives examples of connecting Arts to Mathematics.

Note to Facilitator : Following content can be used by the facilitator during the session

Recreational Mathematics:

Recreational Mathematics is Mathematics that is fun and used as either as a diversion from serious Mathematics or as a way of making serious Mathematics understandable. It is a term referring to Mathematical puzzles, games, riddles and fun-filled activities.

Mathematics puzzles and riddles are fun and interesting, and they help improve problem solving skills and thinking capacity. Puzzles and riddles are also an important area of research for many mathematicians. There are hundreds of puzzle and riddle books dating back to the 1800s.

In recreational Mathematics the *answer* is not the important point. Much more important is the *process* used to find the answer. For one thing, the process can often be

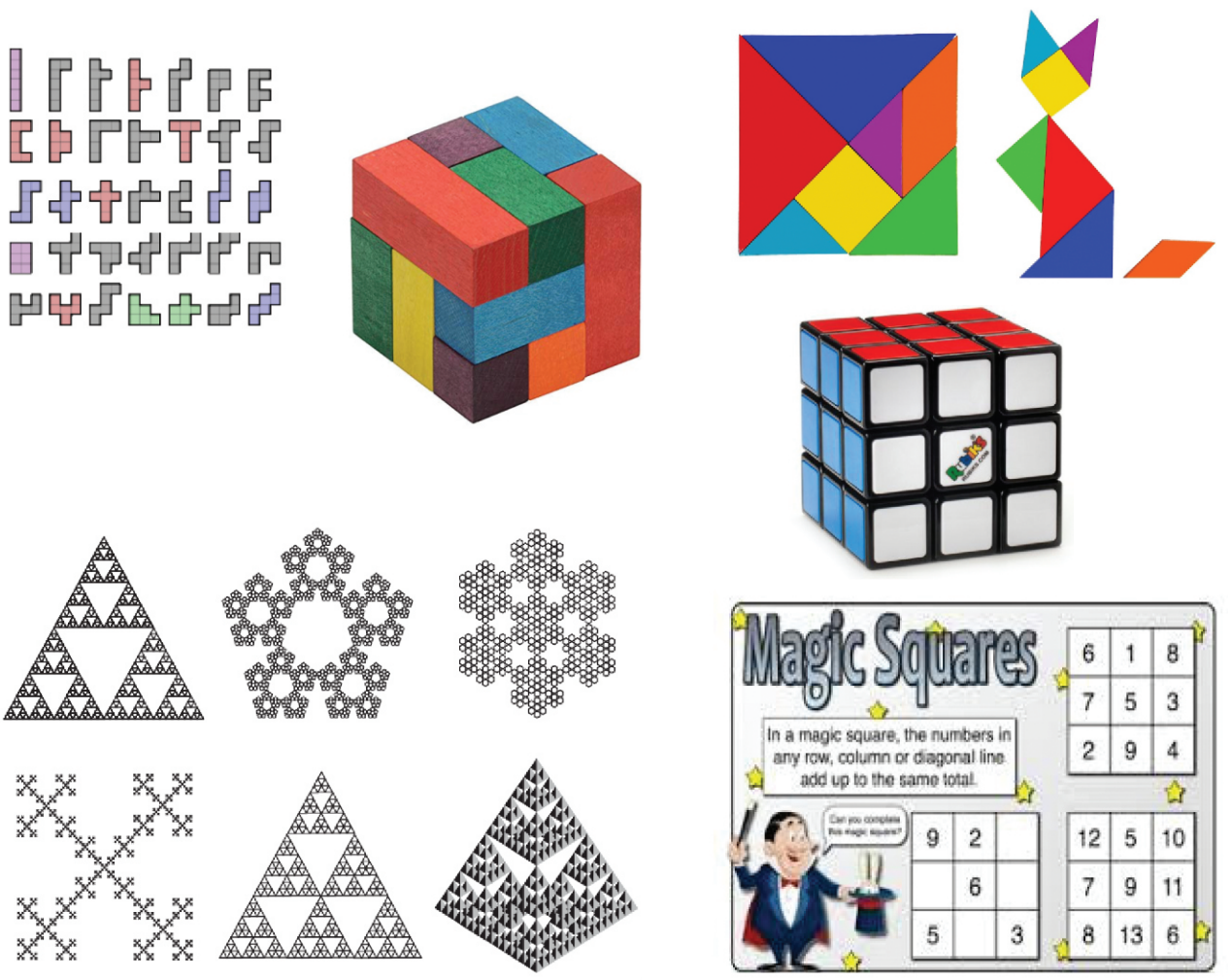
reused to solve other problems. Also, careful examination of the process not only can provide a proof that the answer is correct but also can lead to the discovery of deeper mathematical truths.

Example 1: The placement of the grains in chess board: 1 grain is placed on the first square, 2 grains on the second, 4 on the third, 8 on the fourth, and so on, doubling the number for each successive square. How many grains will be placed in all 64 squares of a chess board?

The resulting number of grains is $2^{64} - 1$, or 18,446,744,073,709,551,615.

Example 2: A father left 17 camels as an asset for his three daughters. When the father passed away, his daughters opened up the will. The will of the father stated that the eldest daughter should get $\frac{1}{2}$ of 17 Camels, the middle daughter should be given $\frac{1}{3}$ rd of 17 Camels, and youngest daughter should be given $\frac{1}{9}$ th of the 17 Camels. How can this be done?

Example 3: Few examples which can be covered under the umbrella of Mathematical recreations are given below. The participants may identify and give inputs.



Alternatively the facilitator can also give examples of the following :

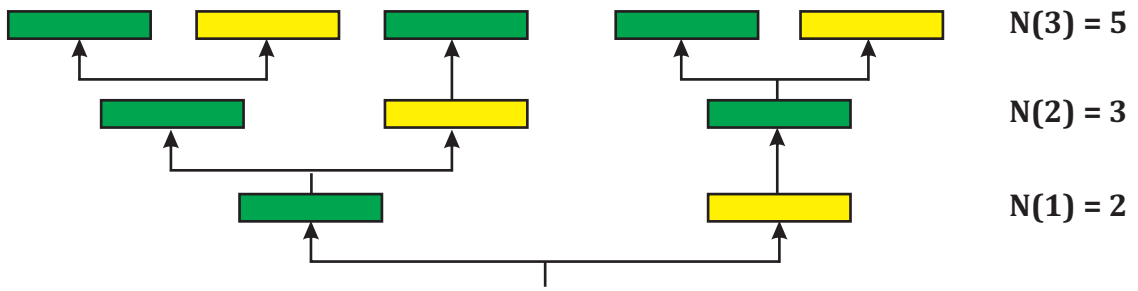
- Polyominoes
- Tangrams
- Fractals
- Platonic Solids
- Soma Cube
- Rubik Cube
- Magic Squares
- Fibonacci Numbers
- Paper Folding
- Sudoku
- Tower of Hanoi
- Shapes and Symmetry
- Mobius Strip

Example -4

There was a Professor living on the first floor of an apartment. The first floor can be reached by walking up a stair consisting of 20 steps. The professor wants to paint the staircase with two colors – Green and Yellow.

Every step will be either painted yellow or green and moreover the professor does not want to have two yellow steps directly after each other. In how many ways can the professor paint his stairs?

Hint: - As specified in the puzzle – the professor has to paint with two colors and after each yellow step there will be a green step and after each green step there will be either a yellow or a green step.



If the stairs have two steps, there are three possibilities.

If the stairs have three steps, there are five possibilities.

Let N(x) be the number of ways the professor can paint a stairs of x steps then step of a stair with x steps can be painted green in N(x-1) ways. In the same way, Professor can paint a stairs of x steps in N(x) = N(x-1) + N (x -2) ways.

In this way the pattern of Fibonacci number can be explained.

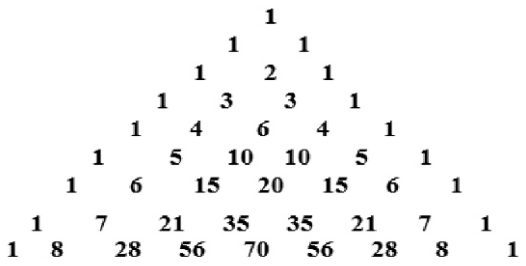
Example-5

When you add, don't go mad
if the number turn around.
The sum will be same
this is a turn around game.

Topic :- ? ? ?

Answer :- Commulative law

Example-6



Fibonacci number which was earlier given by Hemchandra, concept of negative number, concept of infinity are all India's contribution.

The concept of writing big numbers using decimal system was first introduced by Aryabhata. He was the first Indian to give the value of Pi correct up to 4 decimal places.

In Lalit Vistara, a book related to Mahatma Buddha's life describes that Siddhartha, could name number in the multiple of 10 up to 10^{421} which is bigger than 10^{100} known as googol.

Brahmagupta was the first who developed the operation rule of zero such as –

A + 0 = A ; A – 0 = A and A x 0 = 0

Srinivasa Ramanujan had given the formula to derive the value of pi which is now being used to derive the billions of digits of pi.

Example-8

Some question for discussion

1. Why do we read 22.48 as twenty-two decimal four eight?
2. Why is negative times negative a positive?
3. Why 0.999 . . . . . = 1?
4. What is 0 to 0 power?
5. Why is 0/0 is indeterminate and 1/0 is undefined?
6. In division of a fraction, why the fraction at the right side get reversed and multiplication sign replaces division sign?
7. Why do we take pi equal to 22/7 and sometimes 3.14? Who introduced these values in Mathematics?
8. What is the correct order of operations? What is the difference between BODMAS and PEMDAS?
9. While dividing a line segment or an angle we take an arc length more than half, what is its reason?

SESSION 6

Learning from History of Mathematics

(Duration : 45 minutes)

Objectives:

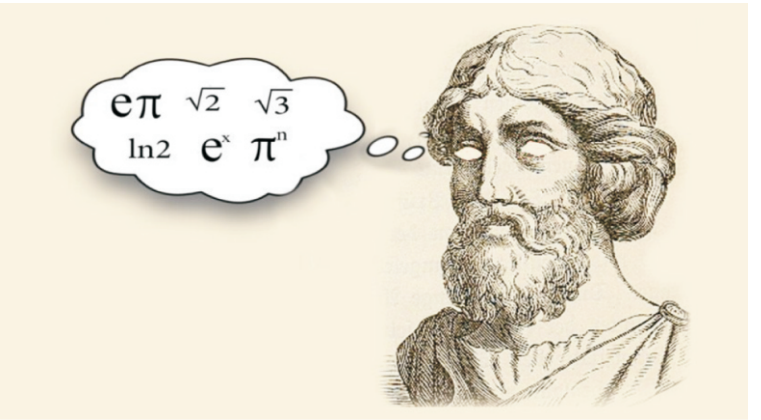
At the end of the activity, the participants will be able to:

- appreciate the use of History in teaching mathematical concepts
- give some examples of use of History in teaching mathematical concepts
- use development of mathematical ideas in the past for teaching and learning.

Preparation: PPT with the history of emergence of rational numbers

Procedure:

1. Facilitator present the history of conflict between the idea that there is no irrational numbers and the existence of irrational number by narrating the letter written by Hippausus 21st July , 540 BCE. (Refer the box below). Facilitator should present the letter with proper intonation and dramatize the same.
2. Facilitator asks the participants to provide alternate proofs of showing that square root of 2 is irrational.
3. Now participants are asked to give some examples of history to be used in teaching mathematical concepts. Story telling techniques (with intonation, gestures etc) should be used for narration. **Emphasis should be given on thought process in development of concepts rather than events happened in the past**.
4. Handout 4 which is an extract from the short stories about Numbers written by Shri Rajnish Kumar, is shared. The participants are asked about the use of this content in classroom teaching of trigonometry.



Hippasus of Metapontum is credited as the first person to discover irrational numbers. However, legend says that he lost his life because of this discovery as the Pythagoreans were not happy with this new development. It went against their belief that all numbers were rational and that there was perfect harmony in the world. Here is the letter written by Hippasus during his stay with the Pythagoreans.

21st July, 540 BCE

Dear Reader,

I write this letter in haste while I still can. The truth needs to be known and I may not live to tell it. I saw it in HIS eyes. HE was furious. Everything belongs to HIM—that's what we, Pythagoreans have always believed but HE does not want to own something he can not explain. I have tilted the balance and destroyed the harmony, according to them. I will be made to pay for it.

Before that, I need to tell you that we, the Pythagoreans, don't know everything. There are mysteries in the universe that we cannot understand. HE does not want to accept that.

We said all numbers could be represented as a fraction. The square root of a number is a number too. So, "2" is a number. This means it can be written as a fraction, i.e. in the form of a/b. Therefore, in the simplest form,

$$2 = a/b$$

Now if we square both sides, we will get

$$(\sqrt{2})^2 = (a/b)^2$$

$$2 = a^2 / b^2$$

This means,

$$a^2 = 2b^2$$

This equation shows that a^2 is an even number as it is the product of 2 with b^2.

If a^2 is an even number, then 'a' must be an even number too. This is because, an odd number multiplied with an odd number (i.e. itself) will only give an odd number.

This shows that 'a' is an even number. That means, 'a' can be written as,

$$a = 2c$$

That is, 'a' can be written as a product of 2.

Substituting a = 2c in the initial equation gives us,

$$2 = (2c/b)^2$$

$$2 = 4c^2/b^2$$

Thus,

$$b^2 = 2c^2$$

This means that 'b' is an even number too. If both 'a' and 'b' are even numbers, then a/b is not in its simplest form. It can be divided by 2, right? Then this means, that you can not write $\sqrt{2}$ as a fraction, isn't that so?

Then what is $\sqrt{2}$? It doesn't seem to be a number that behaves rationally.

When I mentioned this to HIM, HE opened his eyes wide and stared at me. HE went through my calculations again and again. Then, HE softened his glare and said,

"Let's go for a bout of fresh air in the boat today evening and discuss this further."

I know what that means; I am going to be thrown into the Mediterranean. It is not that I care about my life; but I care about Mathematics. We need to understand that Mathematics has a life of its own. It will always keep growing and there will always be problems beyond our grasp. The quest to conquer this subject is never ending.

I can hear the footsteps. They are coming for me. I will hide this letter. Someday, someone will understand there is more to Mathematics than what we know.

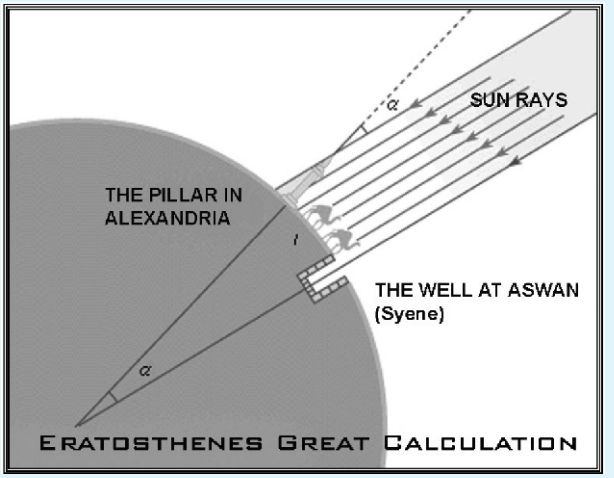
Yours, Hippasus

Handout-3 (Source : Short Stories on numbers – Shri Rajnish Kumar)

CALCULATING SIZE OF EARTH – TRIGONOMETRICAL POTENTIALITIES

Since its earliest days, geometry has been applied to practical problems of measurement—whether to find the height of a pyramid, or the area of a field, or the size of the earth. "Geometry" derives from the Greek *geo* (earth) and *metron* (to measure). But the ambition of the early Greek scientists went even farther; using simple geometry and later trigonometry, they attempted to estimate the size of the *universe*.

In the year 240 B.C. Eratosthenes, a famous Greek scientist achieved the feat for which he is chiefly remembered, *computing the size of the earth*.



It was known that at noon on the day of the summer solstice (the longest day of the year), the sun's rays directly illuminated the bottom of a deep well in the town of Syene (now Aswan) in Upper Egypt. The sun was exactly overhead at noon as Aswan lies on the Tropic of Cancer. The shadow of a vertical rod in Alexandria, due north of Syene, showed that the sun was about 7.2 degrees from the Zenith, this is about one-fiftieth of a full circle (360^{o}).

Eratosthenes very rightly assumed that the sun is so far away from the earth that its rays reach us practically parallel, hence the difference in the sun's elevation as seen from the two locations must be due to ***the sphericity of the earth***. Since the distance between Alexandria and Syene was 5,000 stadia (as measured by the time it took the king's messengers to run between the two cities), the circumference of the earth must be fifty times this distance, or 250,000 stadia.

The exact length of the stadium, the geographical distance unit in the Greek era, is not known; estimates vary from 607 to 738 feet, the smaller figure referring to the Roman stadium of later use. The circumference of the earth as found by Eratosthenes is therefore between 29,000 and 35,000 miles. The correct value is amazing close 24,818

miles for the polar circumference and 24,902 miles for the equatorial. Eratosthenes used the science of geometry in its literal sense: ***to measure the earth***.

Barnabas Hughes, in his *Introduction to Regiomontanus' On Triangles* said,

> *It is quite difficult to describe with certainty the beginning of trigonometry. In general, one may say that the emphasis was placed first on astronomy, then shifted to spherical trigonometry, and finally moved on to plane trigonometry*.

Now a basic query, how did this word ***sine*** originate, there are several stories told in different ways, but what is found to be most authentic was an early Hindu work on astronomy, the ***Surya Siddhanta*** gives a table of half-chords based on Ptolemy's table. The sine as a function of an angle was first described in the *Aryabhatiya* of *Aryabhata* (ca. 510), considered the earliest Hindu treatise on pure mathematics. In this work *Aryabhata* II (also known as *Aryabhata* the elder; born 475 or 476, died ca. 550) uses the word *ardha-jya* for the half-chord which is shortened to *jya* or *jiva*.

The etymological journey of the modern word ***"sine"*** is interesting and starts from here. When the Arabs translated the *Aryabhatiya* into their own language, they retained the word *jiva* without translating its meaning. In Arabic and Hebrew, words consist mostly of consonants, the pronunciation of the missing vowels being understood through common usage. Thus *jiva* could also be pronounced as *jiba* or *jaib*, and *jaib* in Arabic means bosom, fold, or bay.

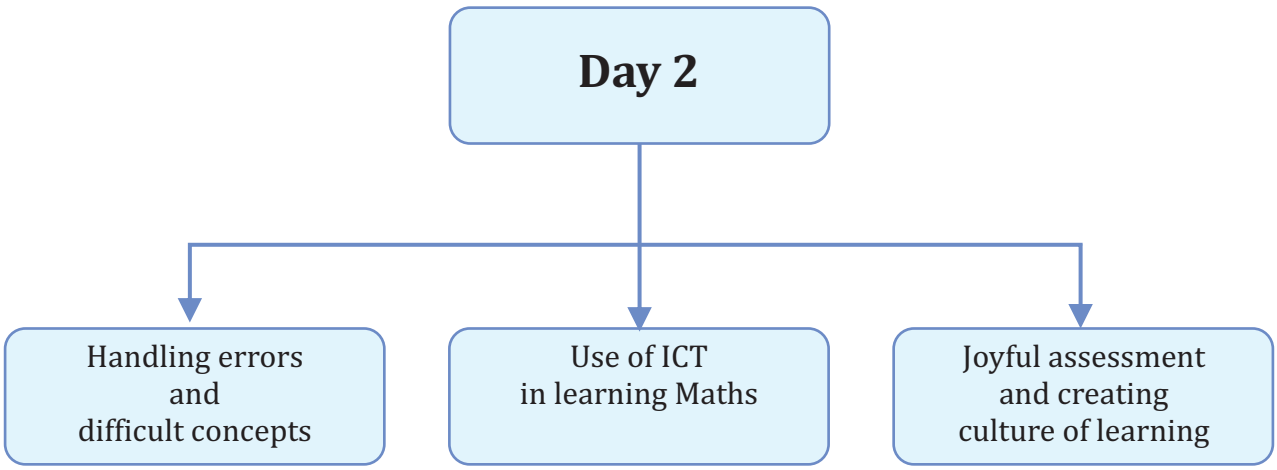
When the Arabic version was translated into Latin, *jaib* was translated into ***sinus***, which means bosom, bay, or curve. Soon the word ***sinus or sine*** in its English version became common in mathematical texts throughout Europe. The abbreviated notation ***sin*** was first used by Edmund Gunter (1581–1626), an English minister who later became professor of astronomy at Gresham College in London. In 1624 he invented a mechanical device, the "Gunter scale" for computing with logarithms - a forerunner of the familiar slide rule - and the notation ***sin*** (as well as ***tan***) first appeared in a drawing describing his invention.

The remaining five trigonometric functions have a more recent history. The cosine function, which we regard today as equal in importance to the sine, first arose from the need to compute the sine of the complementary angle. Aryabhata called it ***kotijya***. The name ***cosinus*** originated with Edmund Gunter: he wrote co sinus, which was modified to ***cosinus*** by John Newton (1622–1678), a teacher and author of mathematics textbooks (he is unrelated to Isaac Newton) in 1658. The abbreviated notation ***cos*** was

first used in 1674 by Sir Jonas Moore (1617–1679), an English mathematician and surveyor.

The functions secant and cosecant came into being even later. The word "tangent" comes from the Latin ***tangere***, to touch. Its association with the tangent function may have come from the fact that the tangent to a circle is related to this function on the unit circle.

> *The science of trigonometry was in a sense a precursor of the telescope. It brought faraway objects within the compass of measurement and first made it possible for man to penetrate in a quantitative manner the far reaches of space.*
>
> **Stanley L. Jaki, The Relevance of Physics (1966)**


Day 2
Handling errors and difficult concepts
Use of ICT in learning Maths
Joyful assessment and creating culture of learning

SESSION 1

Handling Common Errors

(Duration : 1 hour)

Objectives:

At the end of this activity, the participants will be able to:

- identify common errors
- suggest remedial measures for reducing errors.

Preparation: Power point presentation

Procedure:

1. The facilitator divides the participants into groups
2. Shows presentation on errors and ask each group to identify the errors in a given situation
3. The group explains reasons for such errors and how to minimize them in classrooms.
4. Each group is also asked to present common errors and strategies adopted by them to reduce errors
5. Facilitator consolidates the points and give inputs.

Note to facilitators: Some of the errors by the students are shown to the participants and strategy to reduce the errors may be discussed.

Common Errors in Mathematics

1) Improper use of non-negative square root sign:

Students seem to be under the misconception that 4 = 2. This is incorrect. The symbol stands for the non negative square root. So, the correct statement is 4 = 2. If we want -2 as the square root, we write – 4 = –2 This misconception arises, because, they are also asked to solve $x^2 - 4 = 0$. Clearly, the answer to this is x = 2. The proper working is as under.

$x^2 - 4 = 0$

$\Rightarrow x^2 = 4$

$\Rightarrow x = \pm\sqrt{4} = \pm 2$

In fact, $\sqrt{x^2} = \begin{cases} x, \text{ if } x \geq 0 \\ -x, \text{ if } x < 0 \end{cases}$

2) We know that $\frac{0}{2}=0$. Many students will say $\frac{2}{0}=0$. Remember that division by 0 is undefined. Here is a very good example of the kind of absurd result that we may arrive at if we divide by 0. Consider the algebraic identity:

$$x^2-y^2=(x+y)(x-y) \quad \ldots(1)$$

Put, $x = y$ in (i) we obtain

$$x^2-x^2=(x+x)(x-x) \quad \ldots(2)$$

$$\Rightarrow x(x-x)=(x+x)(x-x) \quad \ldots(3)$$

$$\Rightarrow x=(x+x) \quad \ldots(4)$$

$$\Rightarrow x=2x \quad \ldots(5)$$

$$\Rightarrow 1=2 \quad \ldots(6)$$

We have managed to prove 1 = 2. This is, clearly, an absurd result. Surely, we have made some mistake somewhere. We have divided both sides of equation (3) by x – x, which is actually 0 and we have taken $\frac{0}{0}$ as 1. Remember that you can't divide by 0.

3) Many a time, students decide that parentheses are not needed at certain steps. They fail to understand the importance of parentheses.

Example 1: Square 3x.

INCORRECT	*CORRECT*
$3x^2$	$(3x)^2=(3)^2(x)^2=9x^2$

Example 2: Square –2

INCORRECT	*CORRECT*
$-2^2=-4$	$(-2)^2=4$

Example 3: Solve for x: $\frac{x-1}{2}-\frac{2x+3}{3}=\frac{1}{5}$

INCORRECT	*CORRECT*
$\frac{x-1}{2}-\frac{2x+3}{3}=\frac{1}{5}$ $\Rightarrow \frac{3x-3-4x+6}{6}=\frac{1}{5}$ $\Rightarrow \frac{-x+3}{6}=\frac{1}{5}$ $\Rightarrow -5x+15=6$ $\Rightarrow -5x=-9$ $\Rightarrow x=\frac{9}{5}$	$\frac{x-1}{2}-\frac{2x+3}{3}=\frac{1}{5}$ $\Rightarrow \frac{3(x-1)-2(2x+3)}{6}=\frac{1}{5}$ $\Rightarrow \frac{3x-3-4x-6}{6}=\frac{1}{5}$ $\Rightarrow \frac{-x-9}{6}=\frac{1}{5}$ $\Rightarrow -5x-45=6$ $\Rightarrow x=\frac{-51}{5}$

4) Improper Distribution:

Example 1: Simplify: $3(2x^2-6)$

INCORRECT	*CORRECT*
$3(2x^2-6)=6x^2-6$	$3(2x^2-6)=6x^2-18$

Example 2: Simplify : $2(3x-5)^2$

INCORRECT	*CORRECT*
$2(3x-5)^2=(6x-10)^2=36x^2-120x+100$	$2(3x-5)^2=2(9x^2-30x+25)=18x^2-60x+50$

5) $\because 2(a+b)=2a+2b,$ students get an impression that everything works like this. Here is a whole list, in which this does not work.

INCORRECT	*CORRECT*
$(a+b)^2=a^2+b^2$	$(a+b)^2=a^2+b^2+2ab$
$\sqrt{a+b}=\sqrt{a}+\sqrt{b}$	$\sqrt{a+b}\neq\sqrt{a}+\sqrt{b}$
$\sqrt{a^2+b^2}=\sqrt{a^2}+\sqrt{b^2}=a+b$	$\sqrt{(a+b)^2}=a+b$
$\frac{1}{a+b}=\frac{1}{a}+\frac{1}{b}$	$\frac{1}{a+b}\neq\frac{1}{a}+\frac{1}{b}$
$\sin(x+y)=\sin x+\sin y$	$\sin(x+y)=\sin x\cos y+\cos x\sin y$

6) Cancellation error:

Example 1: Simplify: $\frac{2x^4-x}{x}$

INCORRECT	*CORRECT*
$\frac{2x^4-x}{x}=2x^3-x$	$\frac{2x^4-x}{x}=\frac{x(2x^3-1)}{x}=2x^3-1$

Example 2: Solve : $3x^2 = x$

INCORRECT	*CORRECT*
$3x^2 = x$ $\Rightarrow 3x = 1$ $\Rightarrow x = \frac{1}{3}$ *(Here, they divide both sides by x without realizing that x could be 0 also.)*	$3x^2 = x$ $\Rightarrow 3x^2 - x = 0$ $\Rightarrow x(3x-1) = 0$ $\Rightarrow$ either $x = 0$ or $3x - 1 = 0$. *In the first case we got x = 0 and in the second case, we get* $x = \frac{1}{3}$ *Both satisfy the equation, hence are solutions.*

7) Improper use of powers of trigonometric functions:

INCORRECT	*CORRECT*
$\sin^n x = \sin x^n$	$\sin^n x = (\sin x)^n$

8) Incorrect use of trigonometric results:

INCORRECT	*CORRECT*
$\cos 85° = \sin(90° - 5°) = \sin 5°$	$\cos 85° = \cos(90° - 5°) = \sin 5°$ *or*, $\cos 85° = \sin(90° - 85°) = \sin 5°$

9) The following is a mistake largely on the part of authors and the mistake is carried over to the students.

INCORRECT	*CORRECT*
Area of the rectangle having length and breadth 6 cm and 5 cm respectively $= 6 \times 5 = 30 cm^2$	*Area of the rectangle having length and breadth 6 cm and 5 cm respectively* $= (6 \times 5) cm^2 = 30 cm^2$

Area of a rectangle has to be expressed in some square units

SESSION 2

Handling Difficult Concepts

(Duration : 1 hour)

Objectives:

At the end of the session, the participants will be able to:

- identify difficult concepts in Mathematics
- develop Self Learning Material in handling difficult concepts
- design strategies for handling difficult concepts through peer group discussion.

Preparation: NCERT textbooks of classes VI- X

Procedure:

1. Facilitator divides the participants in groups and distributes one NCERT text book to each group
2. Group notes down the difficult concepts and pinpoints the difficulties.
3. Facilitator explains the development of Self learning Material(SLM) as one of the strategies for handling difficult concepts.
4. Then each group shares the strategies (including SLM) for addressing these difficult concepts using the learning so far.

Inputs for Facilitator:

Some difficult concepts and how to handle them.

1. Most of the students feel that numbers of the type $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, are the only irrational numbers. Do you agree or not? If not, suggest some more type of irrational numbers, Like, 1.3010015……., π, e, $\frac{1+\sqrt{5}}{2}$ etc.
2. Write the zeroes, roots and solutions corresponding to x^2 mentioning the use of polynomial or equation, whichever is more appropriate solution.
   (i) Zero(s) of polynomial x^2 is x= 0 (one zero)
   (ii) Roots of equation $x^2 = 0$ are x= 0, 0 (Two identical roots)
   (iii) Solution of equation $x^2 = 0$ are its real zeroes. i.e., x = 0

(iv) Solution of $y = x^2$ are coordinates of any point lying on its graph. For example, coordinates (0,0), (1,1), (-1,1) are some of the solutions. In fact, it has infinite solutions.

3. The result

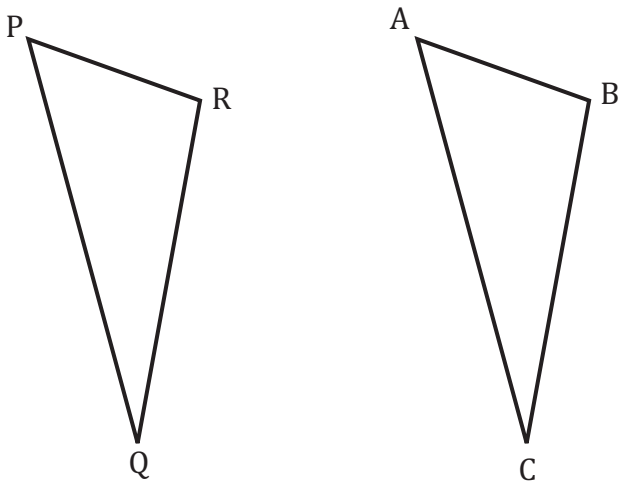
$HCF \times LCM = \text{Product of the numbers}$

Above statement is valid for how many numbers?

$HCF\ (m, n) \times LCM\ (m, n) = mn$

$HCF\ (m, n, r) \times LCM\ (m, n, r) \neq mnr$

4. Does $\Delta PQR \cong ACB$ mean $\Delta PQR \cong \Delta ABC$?



No.

As when we say that

$\Delta PQR \sim \Delta ACB$, it means $\angle P = \angle A$, $\angle Q = \angle C$ and $\angle R = \angle B$

Handout-4

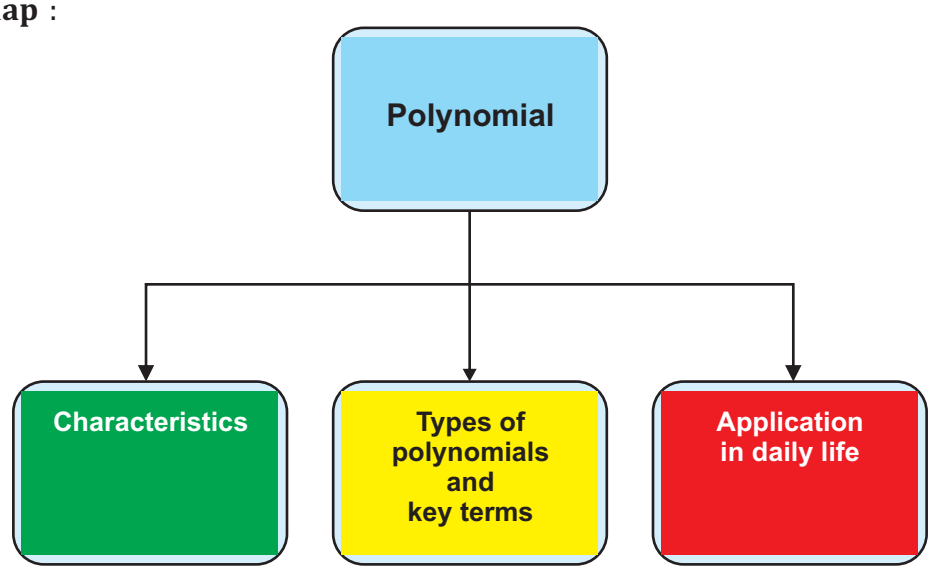
A model Self Learning Material developed on Polynomial with single variable:

Objectives:

This unit will help the learner to :

- express some life situations in mathematical language using polynomials
- appreciate the importance of polynomials in day to day life
- express the key terminologies of a polynomial
- solve mathematical problems using polynomials

Concept Map :



The word 'Polynomial' is derived from two words poly and nomial : poly- (meaning "many") and nomial (in this case meaning "term"). You may wonder how learning polynomials will be helpful in our life. See what Seema wants to achieve and how can you help her using the concept of polynomial.

Seema is an onlne taxi driver and she has set a target to earn **Rs. 1000** on Monday. The meter charges the customer at a **rate of Rs 10 per kilometer** for travelling. But the driver **gets only half of the amount paid** by the traveller after deduction of the commission and other charges by the taxi company.

Challenge 1 :

How many kilometre should she drive to achieve her target on Monday?

Solution :

If x is the distance covered by her in km, she will earn an amount equals to the product of rate of earning per km (10/2) and distance in km (x) .

$$\text{i.e } (10/2)\, x = 5x$$

If she wants to earn Rs. 1000, then 5x must be equated to 1000

$$\text{i.e } 5x = 1000$$

Therefore value of x = 200 km. Hence, she should drive 200 km on Monday to earn Rs. 1000.

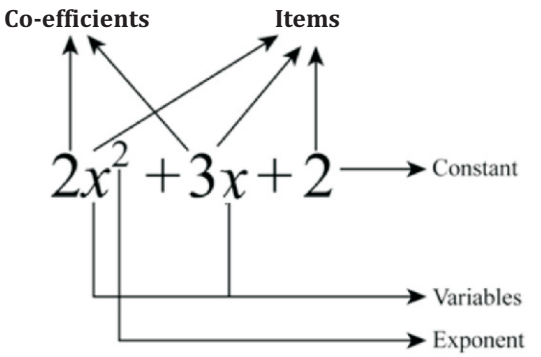
PS : See how her target will change if she wishes to earn less or more (say Rs. 500 or Rs. 2000) money :

- If she wants to get Rs. 500 then she should equate 5x =500 and calculate her target as 100 Km
- If she wants to get Rs. 2000 then she should equate 5x =2000 and calculate her target as 400km.

Here we have expressed a real life problem into mathematical language using 5x to solve it. In this case **5x is called a polynomial in the variable x**. Some other examples of polynomials are :

$$x^2,\quad 2x+3,\quad 2x^2+3y+2.$$

What are the components of a Polynomial?



$2x^2$, 3y and 2 are parts of the polynomial and are called **terms** of a polynomial. Thus a **poly_nomial** has **many_terms.**
Power of variable in a polynomial is called **exponent**
The number multiplied to variable is called a **coefficient**

How do you know that an algebraic expression is a polynomial ?

Recall your understanding of an algebraic expression. You can easily see that **polynomials are also algebraic expressions**. Then what are the characteristics of a polynomial ?

Challenge 2 :

Find out the characteristics of polynomials from the following table and see if you can define a polynomial.

Polynomial	Algebraic expression which are not polynomial
$2x$	$2x^{-1}$
$3x$	$3x^{-1/2}$
z^2+1	z^{-1}
X^3+X^2+3	$z^{-1}+1$
2	$y^{-1}+1$
$p+2$	$\sqrt{3x^{-2}}$

What is a Polynomial?

Algebraic expressions with whole numbers as exponents of variables are termed as polynomials.

That means : if the exponents are -1, $\sqrt{2}$, $\sqrt{5}$, 1/3 etc (any fraction or negative number) then such an algebraic expression cannot be considered as a polynomial. In short they are the simplest forms of algebraic expressions where exponents are only whole numbers.

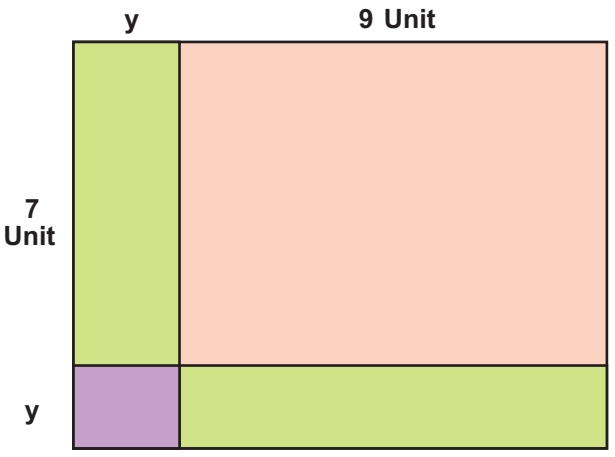
Challenge 3 :

Give new examples of the following

Sr. No.	Definition and example	Write another example
1	Polynomials having only one term are called **monomials** Example : 2y	
2	Polynomials having only two terms are called **binomials** Example :, 2x+1	
3	A number is a **constant polynomial** Example : 3, -1	
4	Highest power of the variable in a polynomial is called the **degree** of the polynomial. Example : degree of $2x^3+ x^2+3$ is 3 and degree of x^2+3 is 2	

Challenge 4 :

Write two different polynomials that describe the area of the garden shown on the right. For one expression, think of the rectangle as one large figure, and for the other expression, think of the rectangle as the sum of 4 different rectangles.



Solution :

First Method:

First, we will define the polynomial that describes the area of the rectangle as one figure.

We are tasked with writing an expressions for the area of the figure above. The area of a rectangle is given as A = lb.

We need to consider the whole figure in our dimensions. Let us write the polynomials :

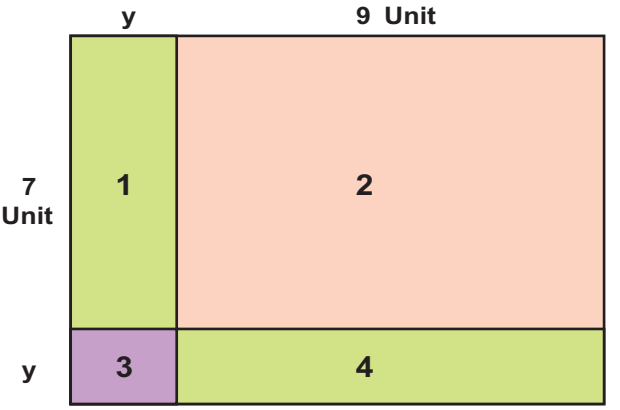
$$l = (y+7) \quad \text{and} \quad b = (y+9)$$

We can multiply these two polynomials to get the area as under :

$$\text{Area} = (y+7)(y+9) = y^2+16y+63$$

Second Method:

Now we will find an expression for the area of the whole figure as comprised by the areas of the four rectangles added together.



Let us write down the polynomials corresponding to the area of each rectangle . Area of each part can be expressed as under:

$$A_1 = 7\ y$$

$$A_2 = 7*9 = 63$$

$$A_3 = y\ *y = y^2$$

$$A_4 = y*9 = 9y$$

We have four polynomials $7y$, 63, y^2 and $9y$

Just see whether we will get same area as we have calculated earlier, if we add these polynomials .

In the above example we have written the polynomials : 7y , 63, y^2, 9y on adding those, we have $y^2+16y+63$

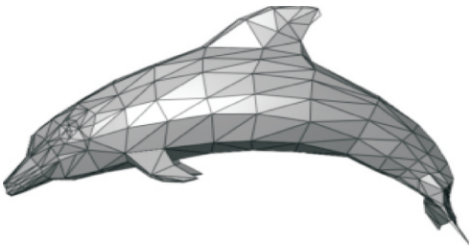
General form of a Polynomial with 'n' degree :

We can write a general form a of polynomial with n degree as under:

$$a_nx^n + a_{n-1}x^{n-1} + \ldots\ldots\ldots\ldots a_1x+a_0$$

where a_n is the coefficient of $(n+1)^{th}$ variable, a_{n-1} is the coefficient of n^{th} term and so on.

Did you know?



(Source: courses.lumenlearning.com)

The dolphin in the image was created by plotting points in space to create connected triangles. This method of rendering graphics works well and is in wide use, but it takes a lot of computer memory. Recently, researchers have been investigating the use of polynomials for rendering graphics in part because it demands less memory in the process. In this process, the surfaces that are rendered are made from solutions to algebraic polynomials.

Challenge 5 :

Find out another concrete example of polynomial used in day to day life. Also write down the degree and type of the polynomial .

Or

Find out real life application of the polynomials : x^2, $2x+3$, $2x^2+3y+2$.

Conclusion :

You need to know that Polynomials are important part of the "language" of mathematics. They are used in nearly every field of mathematics to express numbers as a result of mathematical operations. Polynomials are also "building blocks" in other types of mathematical expressions, such as rational expressions.

Homework :

Find out an example of multivariable polynomial , which you use in day to day life.

One Solution :

You've probably used a polynomial in your head more than once when shopping.

For example, you might want to know how much , three kg of flour, two dozen eggs and three packets of milk cost. Before you check the prices, construct a simple polynomial, letting "f" denote the price of flour, "e" denote the price of a dozen eggs and "m" the price of a packet of milk. It looks like this: 3f + 2e + 3m.

This basic algebraic expression is now ready for you to input prices.

If flour costs Rs. 50, eggs cost Rs 20 a dozen and milk costs Rs. 20 a packet , you will be charged 3(50) + 2(20) + 3(20) = Rs. 250 /-

Reference:

https://courses.lumenlearning.com/boundless-algebra/chapter/introduction-to-polynomials/

https://sciencing.com/everyday-use-polynomials-6319219.html

Note to teacher; Now ask students to Solve NCERT exercise 2.1 . Please reflect on whether contextualization has reduced Mathematical rigour which is there in NCERT text book .

SESSION 3 & 4

Using IT in Teaching Mathematics

(Duration : 2:45 minutes)

Objectives :

At the end of the session, the participants will be able to:

- learn GeoGebra and its applications
- identify online tools for teaching and learning mathematics.
- Share various online resources to the group

Preparation: Installing software in computers and preparation of computer lab

Procedure:

1. Facilitator takes the participants to the computer lab
2. Conducts hands on with GeoGebra linking to different concepts
3. Ask the participants to create a learning material using GeoGebra
4. Facilitator also shares the platforms like Khan academy , Kahoot etc.

Inputs for facilitator:

"Technology can greatly aid the process of mathematical exploration". The potential of computers and computer software should be exploited to make "school mathematics enjoyable and meaningful".

Research studies have shown that the appropriate use of technology can transform the mathematics classroom into a dynamic interactive learning environment where the learner can develop a deeper understanding of the subject rather than focusing only on practicing manipulative skills. Open source software GeoGebra (www.geogebra.org) is a powerful tool, which can enhance understanding of many concepts with the visualization and exploration. In addition to GeoGebra (which is an online and offline application) there are number of websites and online resources; these resources may be used in the classroom as demonstrative as well as exploratory tool in teaching and learning of mathematics.

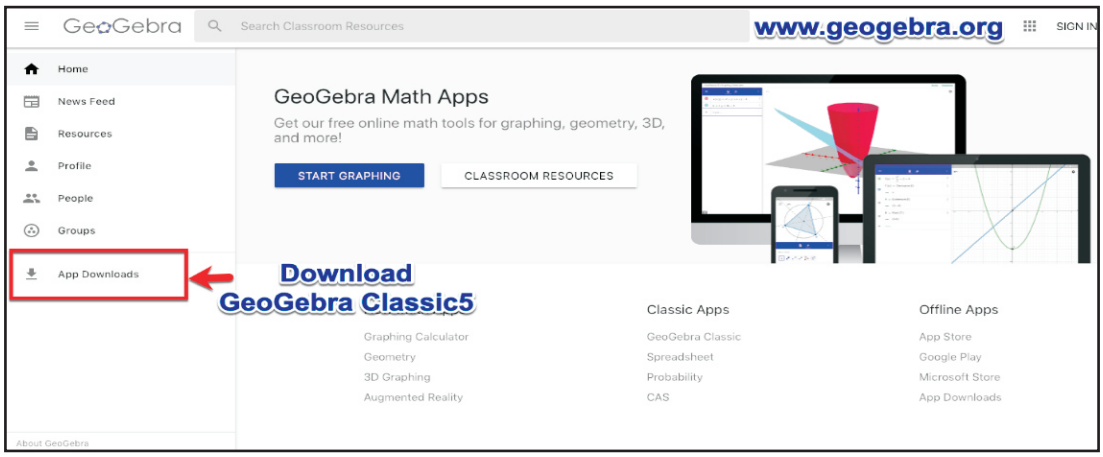
GeoGebra

(Source: www.geogebra.org)

About GeoGebra:

GeoGebra is dynamic mathematics software for all levels of education that brings together geometry, algebra, spreadsheets, graphing, statistics and calculus in one easy-to-use package. GeoGebra has become the leading provider of dynamic mathematics software, supporting Science, Technology, Engineering and Mathematics (STEM) education and innovations in teaching and learning worldwide.

Tasks in GeoGebra may be designed to enable students to make conjectures and understand mathematical concepts. Multiple representations allow the learner to visualize and explore concepts, which are otherwise difficult and abstract. Translated by teams of volunteers, GeoGebra is available in over 60 languages (including Hindi and Tamil). It is a multi-platform application, available either through a web browser or as a stand-alone downloadable application that works across most devices. To work offline, **GeoGebra Classic 5** can be downloaded from the website www.geogebra.org. Users can also download the apps (from Google Play Store or iOS app store) on their mobile phones for easy accessibility.



Learn how to use GeoGebra:

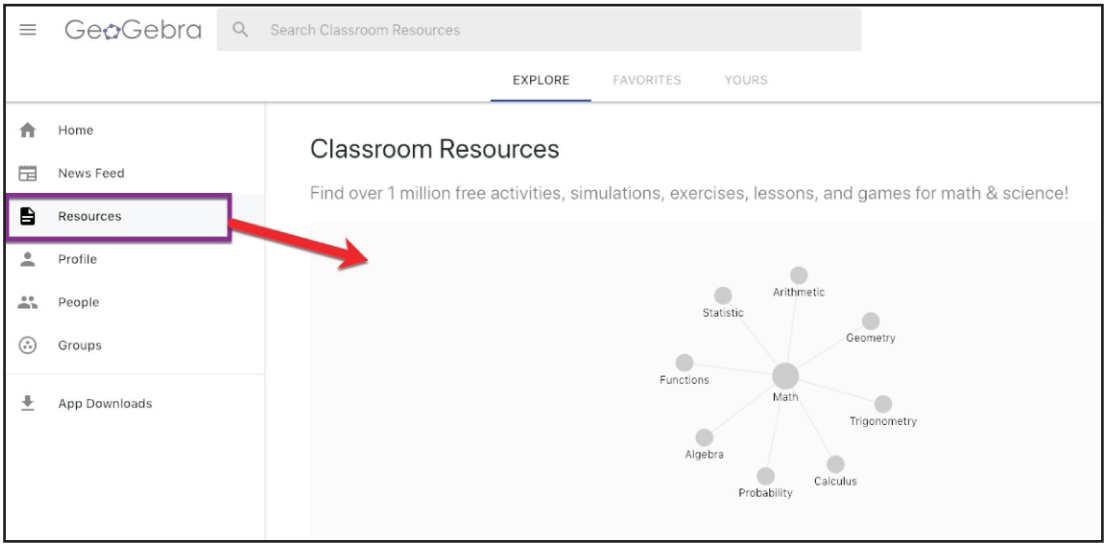
For a beginner following resources will be helpful in getting started.

- Learn GeoGebra in 15 minutes with one of the self-learning courses available at this link: https://www.geogebra.org/m/Ebm5wBW5
- For a more detailed course follow the link: https://www.geogebra.org/m/XUv5mXTm
- GeoGebra Geometry App: Beginner Tutorials with Lesson Ideas: https://www.geogebra.org/m/NUtDnGgC

- Learn how to create Dynamic Activities using the Online Editor provided on the GeoGebra Materials Platform: https://www.geogebra.org/m/deff7B2d

Ready to Use Resources:

GeoGebra website has more than a million Classroom Resources mapped to Arithmetic, Geometry, Trigonometry, Algebra, Calculus, Probability, Functions and Statistics. Each topic is further subdivided into multiple sub topics. These resources are free activities, simulations, lessons and games which can be used as they are or can be downloaded, edited and customized by the teacher.



Here are some of the resources available on the website which may be used to showcase the possibilities with GeoGebra.

In addition to the ready to use GeoGebra applets available at the GeoGebra website, teachers and students are advised to create and explore the applets. Geometry of classes 6-10 can be made interesting and exploratory with GeoGebra

Activity 9 (1) - Angle Sum Property of a Triangle

Objectives: At the end of this activity, the participants will be able to:

Verify the result "Sum of interior angles of a triangle is 180 degrees"

Preparation:

The participants should have access to a laptop/desktop with GeoGebra Classic 5 software installed and ready to use. A computer lab with laptops /desktop for each participant will be a preferred setup.

Procedure:

The facilitator will start by giving an introduction to the software (content given above can be referred to).

Participants will be asked to identify the icon for GeoGebra  and double click to open a new file. Facilitator will give a basic tour of the tool bar and participants will try the tools for Point, Line, Conics, etc.

Once the participants have experienced the possibilities the facilitator will lead them step by step through the instructions of **Activity 1** The printed copy of the instructions may be given to the participants but **only after** the activity is completed (participants will get distracted reading the steps from the sheet). The Facilitator will do the steps of construction and ensure that all participants keep up with the same

- **Hands on work and challenges to participants.**

  Participants will create the applet as instructed by the facilitator. Participants may be asked to think of anyone result of Circles (grade 10) and verify using GeoGebra.

- **Give inputs on how to use in the classrooms**

  To use the resource created in Activity 1 with students, suggest asking questions as you drag a vertex: "What do you see changing?" "What remains constant?" What will happen if triangle is an acute angled triangle? What if the triangle is a right-angled triangle? (Change values of angles to specific by dragging the vertices and confirm the conjectures)

Discussion Points:

- What is the advantage of using technology in this activity?
- Which activity will students enjoy more: Paper activity or GeoGebra activity?

Activity 9 (2) - Pythagoras Theorem

Objectives: At the end of this activity, the participants will be able to:

Verify the Pythagoras Theorem

Preparation:

The participants should have access to a laptop/desktop with GeoGebra Classic 5 software installed and ready to use. A computer lab with laptops /desktop for each participant will be a preferred setup.

Procedure:

Facilitator will use this https://ggbm.at/ZFTGX57r applet to demonstrate the visual proof of Pythagoras Theorem

Note for RP: Drag the points in the sketch to match colours. Ask the participants: What do you notice? If the dimensions of triangle are changed by dragging blue points what changes and what remains same? What does this prove? How does this prove it?

Use this opportunity to highlight the possibilities opened by technology in making learning joyful and meaningful. This applet needs advanced GeoGebra skills but simpler constructions can be done for the verification of Pythagoras theorem. The next activity will be step by step construction in which the Facilitator will lead the participants through the step by step instructions given in the handout of **Activity 2**

- **Hands on work and challenges to participants.**

  Participants will create the applet as instructed by the facilitator. Participants may be asked to create an applet where instead of squares any other regular polygon is constructed on the sides of the right triangle. What is the relation of the areas of these regular polygons? Instead of a right angle triangle, start with any triangle; is the result true?

- **Give inputs on how to use in the classrooms**

  To use the resource created in Activity 2 with students, suggest asking questions as you drag a vertex: "What do you see changing?" "What remains constant?" What will happen if triangle is not right-angled triangle?

Discussion Points:

- How do you introduce Pythagoras Theorem to students in class 7?
- What is the advantage of using technology in this activity?
- Which activity will students enjoy more: Paper activity or GeoGebra activity?

Some Other Resources/Suggestions:

Technology can be used for effective assessment and feedback. Here are some web-based tools, which can be used, as and when required. Internet/WiFi with good bandwidth is required.

A. Kahoot: Create a fun learning game in minutes – called 'Kahoots'. You can make a series of multiple-choice questions or try a Jumble. The format and number of questions are entirely up to you. Add videos, images and diagrams to your questions to amplify engagement.

The facilitator can play a game with the participants (participants can use their smartphones to play the game)

Here is one ready to use Kahoot : https://create.kahoot.it/details/name-the-angle/48567209-ca78-4030-9c50-400b7b3ba44d

Here is video to learn how to create a Kahoot : https://youtu.be/AiB3gmSTPog

Create a Kahoot in 5 simple steps:

1. Go to https://kahoot.com, and sign up or sign in to your account.
2. Click "Create!". Pick a game type. You can choose from: Quiz (multiple choice questions), Jumble (put the answers in the right order), Discussion (one question survey without points) or Survey (a poll without points to gather opinions).
3. Once chosen, fill in the details and settings for the kahoot. Optionally, add a video to play in the lobby. Don't forget to add a cover image!
4. Set up questions and answers, and mark one or more correct ones. Tune time limits for hard questions and toggle points on or off. Add an image or YouTube video to the questions. Don't forget that you can also upload questions via spreadsheet.
5. When you're done adding questions, click "Save". Preview, play or share your newly made kahoot! Let's play!

More information will be available in the Resources

A. Google Forms

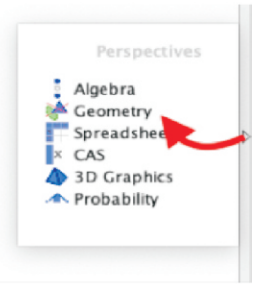
B. Socrative

C. Quizziz

Web Resources:

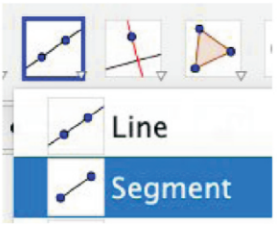
- Interesting activities listed by HBCSE http://mathedu.hbcse.tifr.res.in/activities/
- Hands on Mathematics from Shri Arvind Gupta : http://www.arvindguptatoys.com/math-magic.php
- https://www.khanacademy.org/math/math-for-fun-and-glory

Handout-5



Start with a new file and select the Geometry perspective. To do so, you can go to View and select Graphics or Click on the little arrow on right edge of the screen and select Geometry.

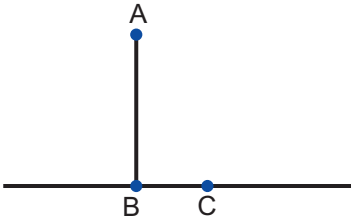
We will first construct a right-angled triangle; have a discussion with learners the possible ways to do so. Following steps are for one of the methods.



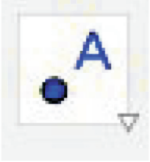
Click the **Segment between two points** tool and click two distinct places on the drawing pad to construct segment ***AB*** .

If the labels of the points are not displayed, click the **Move button**, right click each point and click *Show label* from the context menu.

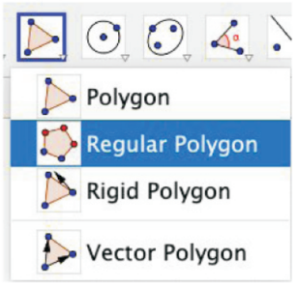
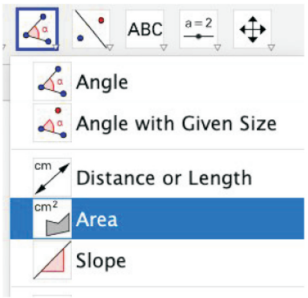
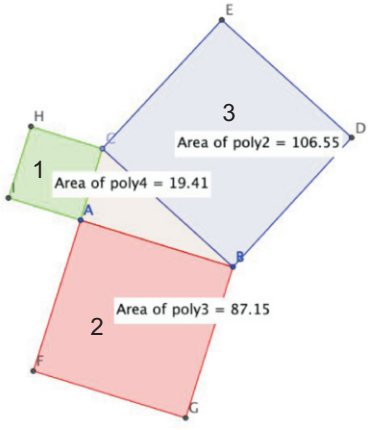
(The *context menu* is the pop-up menu that appears when you right click an object.)

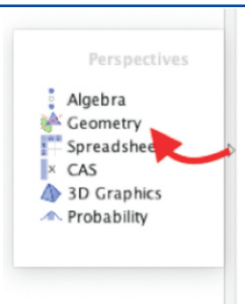
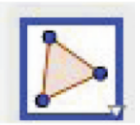
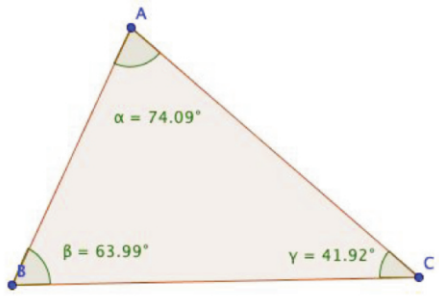


Next, we will construct a line perpendicular to segment ***AB*** and passing through point ***B***. To do this, choose the **Perpendicular line** tool, click segment ***AB***, then click point ***B***.



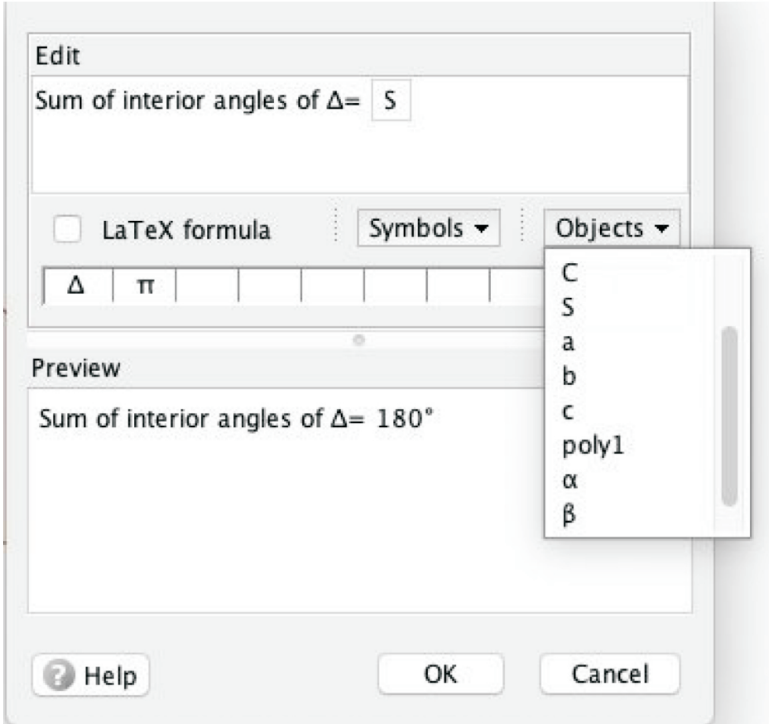
Next, we create point ***C*** on the line. To do this, click the **New point** tool and click somewhere on the line. Your drawing should look like the figure above.

	Hide everything except the three points. To hide the line, right click the line and uncheck *Show Object*. Do this, also, to segment ***AB***. Next, we rename point ***B*** to point ***C*** and vice versa. To rename point ***B*** to ***C***, right click point ***B***, click *Rename* and then type the new name, in this case point ***C***, in the *Rename text* box, then click the **OK** button. Now, rename ***B*** (or B1) to ***C***.
	Next we construct a square with side ***AC.*** Click the **Regular polygon** tool, then click point ***C*** and click point ***A*** .
	In the *Points text box* of the **Regular polygon** tool, type 4 since we are going to create a square.
	With the *Polygon* tool still active, click point ***B*** and click point ***C*** to create a square with side ***BC*** . Similarly, click point ***A*** , then click point ***B*** to create a square with side ***AB.***
	Now, let us reveal the area of the three squares. Select the Area tool from the drop down menu of Angle.
	With the area tool activated, click in the interior of each square 

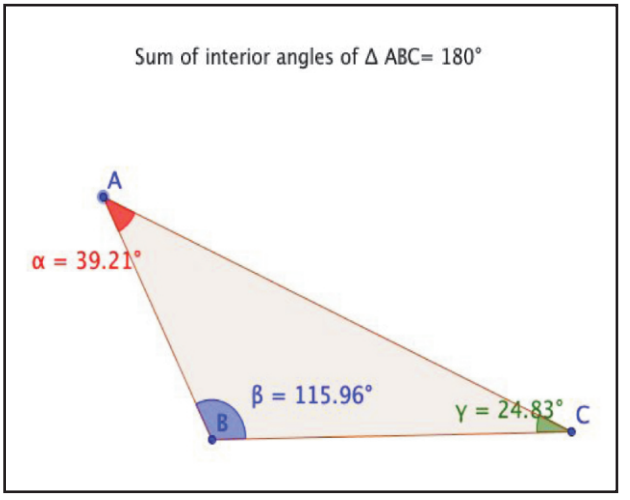
	Move the vertices of the triangle to change the dimensions of the triangle.. What do you observe about the area of the squares? Make a conjecture!
	Start with a new file and select the Geometry perspective. To do so, you can go to View and select Graphics or Click on the little arrow on right edge of the screen and select Geometry
	Click the **Polygon tool** and Create a triangle *ABC (*Use counterclockwise orientation).
	Create the angles *α, β,* and *γ* of triangle *ABC.* To do this, click the **Angle** tool and then click the interior of triangle ***ABC.*** 
Observe the sum of the angles of the triangle ABC.Yes, it is 180 degrees. Will it always be 180 degrees?	
	Use the move tool, to drag any vertex of the triangle ABC. Observe what happens to the measures of the angles. What is the sum of the interior angles for each of the triangles?
Optional: We will change the colors of the angle symbols, angle ***A*** to red, angle ***B*** to blue and angle **C** to green. To change the color of the angle symbol of angle ***A,*** right click the angle symbol (not point ***A***) and click **Properties**. Use Input bar(at the bottom of the screen, if not visible then go to View—>Input Bar) to calculate S = α + β + γ. Note that we will not use the numeric values of the angles, S is calculated for generic values of the angles	

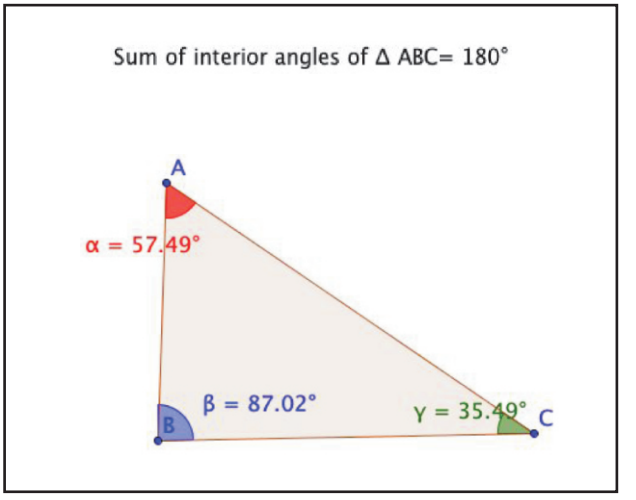
We will now use the Text tool to make a statement: Sum of interior angle of triangle ABC = 1800

Click on the Text Tool to open the dialogue box(as shown below) and type in the statement as shown below. Select S from the drop down list of Objects and to get symbol for triangle, use the drop down list of Symbols.



Drag any of the vertices of triangle and demonstrate that the sum remains 180 degrees.





SESSION 5

Joyful Assessment

(Duration : 45 minutes)

Objectives :

To help the learner to assess himself/herself on solving linear equations for one variable.

Preparation :

- a set of cards with expressions written on them (refer the lesson)
- notebooks/paper
- chalk and blackboard, or markers and whiteboard

Procedure :

1. The facilitator understands the game from the information given on the next page.
2. No information about the purpose is revealed to the participants, by the facilitator.
3. Cards with letters are prepared in advance with equations.
4. Participants pick one card each and find out a partner who has picked the card with same letter.
5. Now the facilitator asks the paired participants to make an equation and solve.
6. After the activity the facilitator, asks the participants, the importance of this kind of assessment and how to find out the difficulties faced by children. Assessment for learning is explained by the facilitator.
7. A PISA based question is discussed with the participants.
8. In the group, the participants are asked to prepare a similar question based on any topic of their choice.

Details of the game:

This game is planned for use with 30 students; however, more cards can be made for play in a larger-sized class. Students might help you to prepare the 30 game cards, or the cards might be prepared in advance. Each card should have a letter of the alphabet (in this case, *A* to *O)* written on it along with a linear expression; there will be two different cards with the same letter and different linear expressions. For example, see the list below. For the letter A there are two cards:

- one card has *A* written on it with the linear expression $4x + 2 - 8x$
- the other card has *A* written on it with the linear expression $3x$. Create additional pairs of cards with the following letters and linear expressions.

Card	Expression On Card 1	Expression On Card 2	Answer
A	$4x + 2 - 8x$	$3x$	$x = 2/7$
B	$6x - 7$	0	$x = 7/6$
C	$7 - 10z$	17	$z = -1$
D	$6x + 16$	$2x - 12$	$x = -7$
E	$6 - 5x$	$13x$	$x = 1/3$
F	$14y + 7$	-6	$y = -13/14$
G	$8x - 4$	-6	$x = -1/4$
H	$7 - 5x$	-10	$x = 17/5$
I	$6x - 17$	$9x$	$x = -17/3$
J	$10x + 7$	17	$x = 1$
K	$20x + 10$	$4 - 10$	$x = -4/5$
L	$15p - 5$	$10p + 10$	$p = 3$
M	$11x + 33$	55	$x = 2$
N	$(6x - 5)/2$	$(3x + 6)/2$	$x = 11/3$
O	$5x - 3x + 7$	$-7 + 8x$	$x = 7/3$

If you have a class of 30 students, shuffle the set of 30 cards and distribute a card to each student. (If you have fewer or more students, shuffle a set of letter cards for each pair of students.) Allow students who get the same alphabet cards to sit together and

solve the equation for the value of the variable. For example, the pair of students who got the two cards with the letter *A* on them will solve for x in the linear equation $4x + 2 - 8x = 3x$

Once students have solved their equations, you might place lettered slips (in this games example, one slip with each letter *A* to *O)* in a bowl or hat. Draw out a slip and read the letter that is written on it. Invite the pair of students who have that letter on their cards to come up to the board to show how they solved their equation. If they do it correctly they win that round of the game.

Assessment

Let all student pairs who correctly solved their equations play another round of the game (with new cards or the same ones). With each repeat of the game, you will eliminate more pairs of students. Play until you have a final winner (a pair of champions). Thus, the game can be used to motivate *and* provide drill in solving linear equations in one variable.

(Source : www.educationworld.com/a_tsl/archives)

Page for creating Slips

Note to facilitator (In advance, the facilitator cuts card s with letters on it)

Expression On Card 1		Expression On Card 2	
A	$4x + 2 - 8x$	A	$3x$
B	$6x - 7$	B	0
C	$7 - 10z$	C	17
D	$6x + 16$	D	$2x - 12$
E	$6 - 5x$	E	$13x$
F	$14y + 7$	F	-6
G	$8x - 4$	G	-6
H	$7 - 5x$	H	-10
I	$6x - 17$	I	$9x$
J	$10x + 7$	J	17
K	$20x + 10$	K	$4 - 10$
L	$15p - 5$	L	$10p + 10$
M	$11x + 33$	M	55
N	$(6x - 5)/2$	N	$(3x + 6)/2$
O	$5x - 3x + 7$	O	$-7 + 8x$

Hand out - 6

PISA sample question :

Mount Fuji is a famous dormant volcano in Japan.

1) Toshi wore a pedometer to count his steps on his walk along the Gotemba trail. His pedometer showed that he walked 22,500 steps on the way up. Estimate Toshi's average step length for his walk up the 9 km Gotemba trail. Give your answer in centimeters (cm)

2) Mount Fuji is only open to the public for climbing from 1 July to 27 August each year. About 200 000 people climb Mount Fuji during this time. On average, about how many people climb Mount Fuji each day?

3) The Gotemba walking trail up Mount Fuji is about 9 kilometers (km) long. Walkers need to return from the 18 km walk by 8 pm. Toshi estimates that he can walk up the mountain at 1.5 kilometers per hour on average, and down at twice that speed. These speeds take into account meal breaks and rest times. Using Toshi's estimated speeds, what is the latest time he can begin his walk so that he can return by 8 pm?

(Source: www.oecd.org/pisa)

SESSION 6

Creating Culture for Mathematics Learning

(Duration : 30 minutes)

Objectives: At the end of the session, the course participants will be able to:

- Share and showcase the work being done in their schools to develop positive culture for learning Mathematics
- Take back concrete ideas to encourage, motivate and excite students to learn Mathematics

Preparation:

The facilitator will be required to keep some examples of the possible ideas readily available to support the session.

Procedure:

1. Facilitators will start the session by inviting participants to share various activities/ initiatives being taken by individual teacher or by the school. He/she will extend the interaction by asking teachers to share the effect of such initiatives.
2. Facilitator will then present his/her ideas .

Some of the suggested ideas are:

a. **Maths Newsletter**: Students can be assigned task to create topic based monthly or twice in a term newsletter for the respective classes. This will encourage students to work in collaboration and research a topic beyond the textbook concepts and bring out creativity in students. Some suggested topics are: Maths in Nature, Patterns in Numbers, History of Mathematics, Mathematics in Architecture etc.

b. **Maths Fair:** A carnival setup may be created to give students a chance to apply logic, mathematical skills, and problem-solving ability to interact and play with Mathematical games. Encourage students to create original games based on mathematical concepts suitable for the age of students who will be invited to play the games. For example, Class IX can create and setup games for Classes VI to VIII to play.

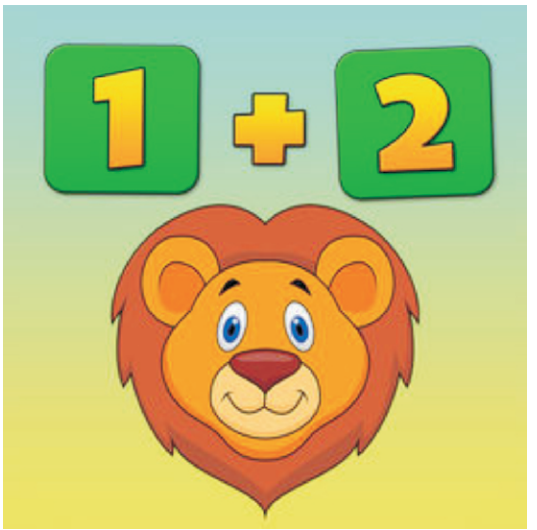
c. **Intra-class and Inter- class Maths Quiz:** Use multiple rounds e.g. Tangram puzzles, identify the mathematician, estimation, some quick calculations etc.

d. **Maths Day:** A day dedicated to Maths is always a great way to create a school with excitement for Maths. All members of school are to be involved in different activities with the day starting with Mathematics Assembly presented by students. Agenda for the

day can be decided by the teachers and concerned authorities. Some of the possible activities are: Lectures by experts, group work on creating mathematical models; fun activities for junior classes may include Rangoli with geometrical shapes, T-shirt painting with mathematical quotations or artwork.

e. **Mathematics Corner/Play Room:** A space dedicated to Maths may be created in school where students can interact with mathematical displays, interactive games and explore mathematics in a non-formal setting. This space may be part of school library or a common area of the school.

f. **Mathematics Assembly :** Once a month student can organize Maths Assembly. Mathematical play, songs, information on history of mathematics can be some of the possible ideas.



Central Board of Secondary Education