EDUCATIONAL PSYCHOLOGIST, *28*(3), 219–233

Theories of Learning and Theories of Development

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Classical Piagetian theory had much to offer education, which could not be derived from the learning theory of its day. Since the cognitive revolution, learning theory has accepted most of Piaget's basic constructivist premises and outstripped his theory in its ability to model the details of children's cognitive structures. Thus, an important question is whether Piagetian theory still has anything distinctive to offer. To support the claim that it does, the notion of a central conceptual structure is introduced. These structures have several features that make their discovery in the context of learning theory unlikely, most notably their generality, the wide range of content domains that they span and their susceptibility to general developmental as well as specific experiential influence. Educational areas in which analysis of central conceptual structures has proven useful include (a) assessment, (b) early childhood education, (c) curriculum design, and (d) remedial instruction.

CLASSICAL THEORIES OF LEARNING AND DEVELOPMENT

As formulated by Piaget, classical developmental theory had much to offer educational psychology which was not derivable from the learning theories of its time. One of Piaget's most important suggestions was that, at several different points in their growth, children acquire new systems of cognitive operations (structures) that radically alter the form of learning of which they are capable. From this it followed that one should begin any educational enterprise by assessing the cognitive structures that are already available to the learner, and one should then present the material one wants to teach in a fashion that can be assimilated by these structures. It also

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followed that one should do everything in one's power to foster the development of children's existing cognitive structures so that they would become more powerful.

A second major contribution of Piaget was his hypothesis concerning the process by which new structures are created. In the context of his theory, children were seen as highly active organisms who construct their own internal structures via a reflexive process. From this it followed that one should rarely, if ever, force learners into a position where they are expected to take a passive role toward the acquisition of their own knowledge. Also, if one wants a curriculum to improve children's intellectual power as well as their conceptual or procedural knowledge, then one should encourage them to explore the limits of their existing structures and to reflect on them.

At the height of its influence, Piaget's theory constituted a powerful driving force behind many attempts at curricular reform, particularly in the areas of science, mathematics, and preschool education (Ginsburg & Opper, 1969, chap. 6; Kamii & Radin, 1967; Karplus, 1964). Two sets of events, however, served to attenuate this influence.

THE COGNITIVE REVOLUTION AND THE BIRTH OF A NEW LEARNING THEORY

The first of these events was that—with the advent of information science—learning theory underwent a profound transformation and became far more cognitive in nature. The result was that learning theory caught up with Piagetian theory in its ability to serve as a basis for educational reform. Nowhere was this change more apparent than in the stance taken by the new learning theorists toward the classical Piagetian propositions. "Postrevolutionary" learning theorists acknowledged that children were active participants in their own learning (Gagné & Briggs, 1979). They also acknowledged that children's systems of cognitive operations might well have a different form of organization early in their development from the form they took later when the domains in which they had to operate were more familiar and the children more "expert" (Chi & Rees, 1983; Resnick, 1983; Simon & Simon, 1978).

As a result of these developments, educators who worked within the new learning framework began to devote a good deal of energy to precisely the sorts of endeavors that Piagetian theory had suggested were necessary, namely (a) assessing the structures and processes of children who were just encountering particular domains of knowledge for the first time and the misconceptions to which these structures and processes gave rise (e.g., Bereiter & Scardamalia, 1985) and (b) generating curricula that took these naive structures into account and challenged children to take an active role in changing them (e.g., Minstrell, 1990; Scardamalia & Bereiter, 1988). Finally, in both of these endeavors, researchers were empowered by the new analytic and technological tools that information theory provided, which far outstripped those that had been available in Piaget's era.

CLASSICAL DEVELOPMENTAL THEORY AND ITS DISCONTENTS

At the same time as classical learning theory was undergoing this revolution, classical developmental theory was beset by major theoretical and empirical difficulties. Nowhere was this more apparent than with regard to the notion of a general system of cognitive operations, which Piaget had characterized with such terms as "operative structure," "logico-mathematical structure" or "structure of the whole" (Piaget, 1970). The existence of these structures was never demonstrated to the satisfaction of developmental psychologists, and their relevance was never demonstrated to the satisfaction of practicing educators. Thus, although most educators would agree that Piaget's historical influence on education was a positive one, fewer and fewer educators looked to his theory as a source of ideas that could be used to guide school or curriculum reform in the modern era, where the list of educational problems includes training children to compete in a global economy, coping with an accelerated pace of technical change, and providing equal opportunity to populations that are becoming increasingly diverse—both from a cultural and an economic standpoint.

ATTEMPTS TO REINVIGORATE THE CLASSICAL DEVELOPMENTAL POSITION

To say that Piaget's classical developmental theory encountered theoretical and empirical challenges is not to say that it lost its vigor completely. The Piagetian tradition continued to have some impact, particularly in the fields of mathematics (Davis, Noddings, & Maher, 1990) and early childhood education. Several interesting theoretical directions were also taken by those who accepted Piaget's basic epistemological assumptions but wished to replace the notion of a "structure of the whole" with some more local structural notion (see the contributions of Carey & Smith, 1993; Strauss, 1993; Wainryb & Turiel, 1993).

The work on which I report here is the indirect outgrowth of one such line of theoretical work that was originated by Pascual-Leone (1969, 1970). Pascual-Leone retained Piaget's view of the human mind as a highly dynamic, self-reflexive system that goes through periods of stability and disequilibrium in its development. However, he replaced the concept of a "structure of the whole" with the notion of a system-wide limitation in mental (M) power (Pascual-Leone, 1969, 1970). As this M-power (or M-space) increased, Pascual-Leone asserted that children could construct more complex responses to any given problem of their choosing. Because each problem had its own unique properties and each child had his or her own response tendencies, cognitive functioning was highly variable across individuals and situations. On the other hand, because all children's functioning at any age was subject to a common ceiling in complexity, there were also certain general properties of young children's thought that were age linked and that transcended any local task (Pascual-Leone, 1969).

As the theoretical structure of Pascual-Leone's (1969, 1970) theory developed, investigators began to draw on many of the same informational concepts as had learning theorists and to integrate them with those proposed by Pascual-Leone. Thus, a wide number of tasks were analysed with regard to the task-specific schemes and misleading factors they possessed (Pascual-Leone, 1970), the "executive strategies" or "control structures" they required (Case, 1974, 1985), and the skills or symbolic systems that resulted (Fischer, 1980; Halford, 1982). Investigators also began to distinguish—as had Piaget before them—between the sort of cognitive restructuring that involves progressive elaboration or extension of existing mental structures and the sort that entails a qualitative transformation of these structures (Case, 1985; Fischer, 1980; Halford, 1982). A new family of stage theories resulted from this endeavor, three of which (Case, 1985; Collis & Biggs, 1982; Fischer, 1980) had the structure illustrated in Figure 1.

As may be seen, four general stages are hypothesized in this family of theories, which are not unlike those postulated by Piaget. At the beginning of each stage, two different sorts of unit (A and B) are integrated and assembled into some qualitatively different form of new unit (A_1–B_1). As children progress through the stage and their working memory increases, they first become capable of differentiating two of these new units (A_1-B_1; A_2-B_2) then apprehending the relation between these units (x) and formulating it explicitly. Finally, as the new systems thus formed become consolidated, the way is paved for transition to a new stage of development in a recursive fashion.

The sort of theory that is illustrated in Figure 1 was applied to education in a variety of ways. One of the most successful of these applications was one that combined the structural postulates of the new developmental theory with the detailed analysis of teaching objectives that had been developed in the learning theory tradition. The result was an instructional technology that proved useful for special and/or remedial education (Case & Bereiter, 1984; Case, Sandieson, & Dennis, 1986).

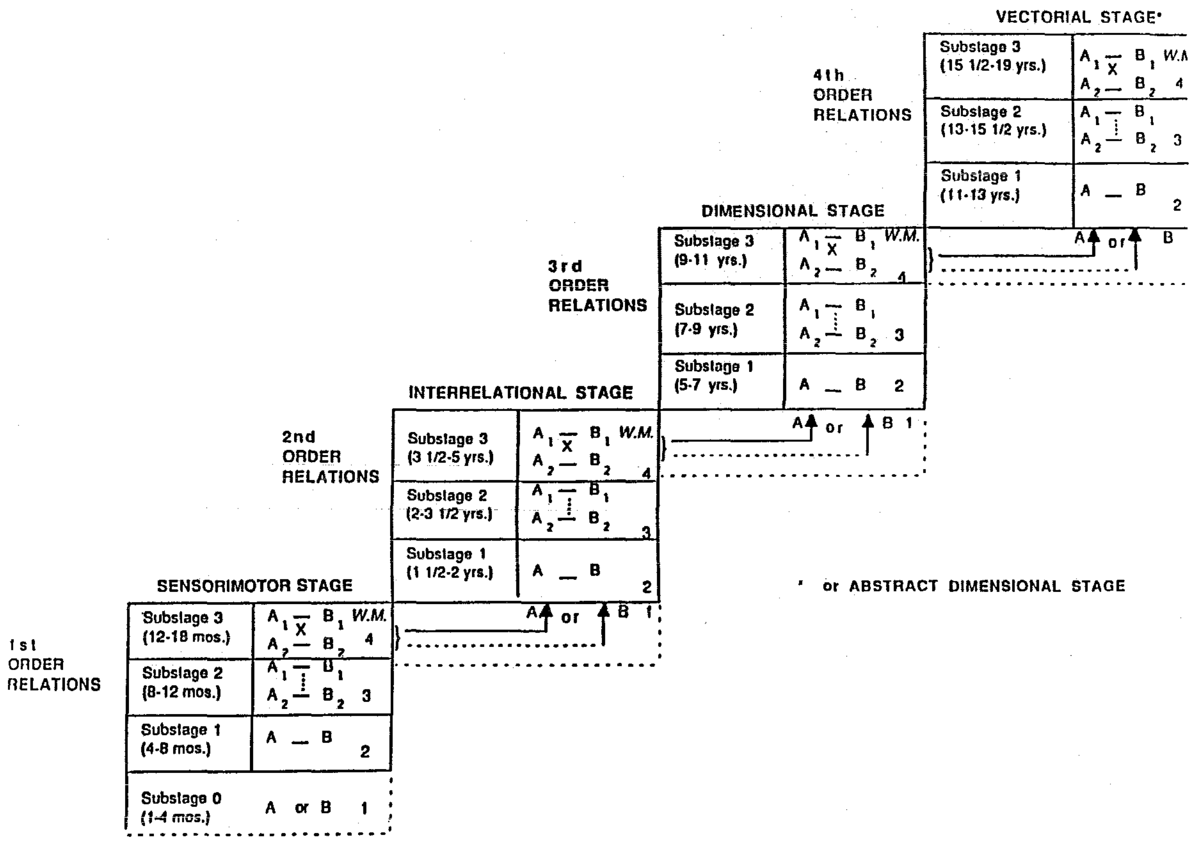

FIGURE 1 General structure of the new stage theories (Case, 1992a; Fischer, 1980).

THE CLASSIC DEVELOPMENTAL HYPOTHESIS REVISITED

In the past 5 years or so, a new proposal has been advanced that promises to give the class of theories represented in Figure 1 additional educational as well as theoretical power. The new proposal resulted from analyzing a wide variety of specific control structures that children apply at different ages—particularly between the ages of 4 and 10 years—and noting that certain groups of these structures share far more in common than would be expected simply on the basis of their common complexity. They also share a common conceptual structure, that is, a common dependence on the same closely related set of central conceptual understandings. The new notion was that, for each of the very broad set of domains investigated by Piaget (e.g., space, number, and causality), children at the various stages indicated in Figure 1 have a small set of *central conceptual structures*. These central conceptual structures are defined as systems of semantic nodes and relations that have a very broad domain of application and play a pivotal role in children's intellectual growth (Case & Griffin, 1990; Case & McKeough, 1990; Case & Sandieson, 1988).

One such structure is the one by which children conceptualize numbers. To illustrate this structure, consider the transition that occurs at about 5 years of age. Prior to this time, children can only make a reasonable prediction about which side of a balance beam will go down as long as they do not have to count the number of weights on each side (Liu, 1981; Marini, 1992). By contrast, 6-year-olds can succeed under these more difficult circumstances (Marini, 1992; Siegler, 1976). A similar trend is seen on a broad array of tasks whose surface structure is quite different. These include problems telling the time (Case, Sandieson, & Dennis, 1986), making change with money (Griffin, Case, & Sandieson, 1992), sight reading music (Capodilupo, 1992), figuring out which of two objects will case a larger shadow (Marini, 1992), and deciding what is the fairest way to distribute a set of rewards (Damon, 1973; Marini, 1992). In each case, 3½- to 5-year-olds approach these problems in a global fashion, counting an array only if this is explicitly requested. In contrast, 5- to 7-year-olds approach the task in a far more differentiated fashion, usually one that involves counting and comparison.

What the revised theory asserts is that this change takes place due to a change in the central conceptual structure that children have available for approaching quantitative problems. Between the ages of 3½ and 5 years children have one structure for dealing with problems of relative magnitude (see Figure 2A) and another for dealing with problems of enumeration (see Figure 2B). Between the ages of 5 and 7 years these two structures become integrated into a single structure of the sort illustrated in Figure 3. The hypothesis is that because 5- and 6-year-olds have this new general

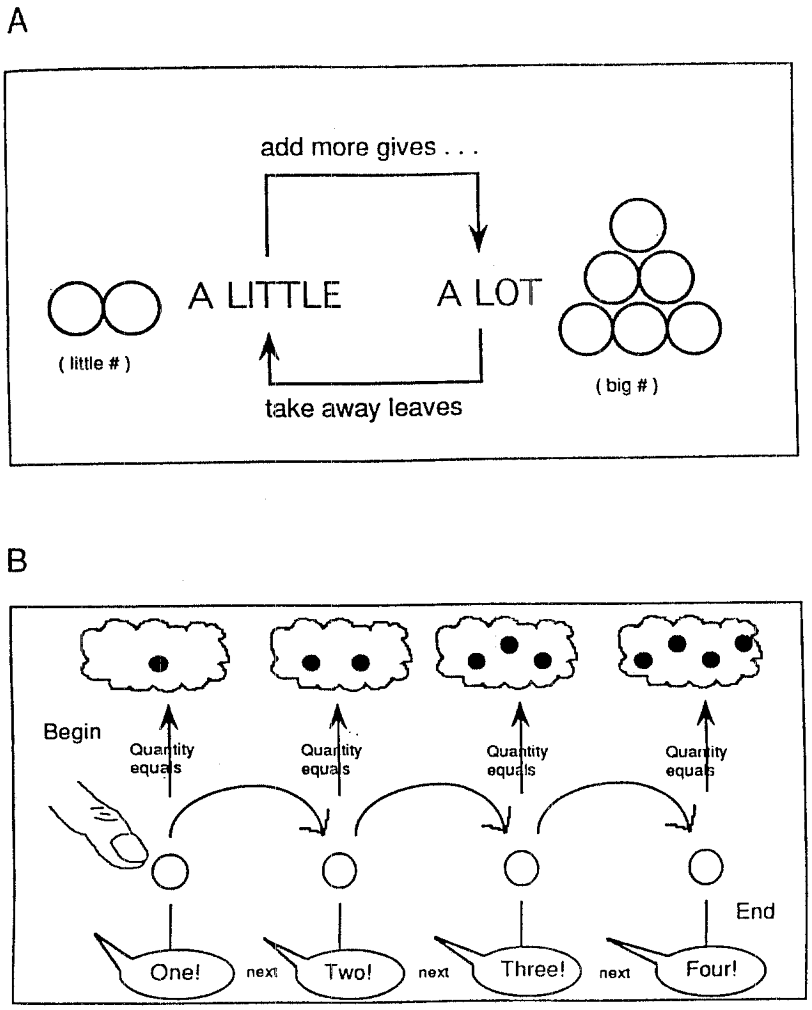

FIGURE 2 A: Schema or "theory" of quantity: the add/take away schema (Resnick, 1983). B: Schema or "theory" of number: the count schema (Gelman, 1978).

structure, they solve all the specific tasks that were mentioned in a different manner.

Between the ages of 5 and 10 years the general structure in Figure 3 becomes automated as children practice its application and experience an increase in functional working memory.[1] Thus, by the age of 8, they can deal with problems that require focusing on two such number lines or

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[1] A maturationally based increase in structural working memory is very probably necessary as well (Case, 1992b; Pascual-Leone, 1988).

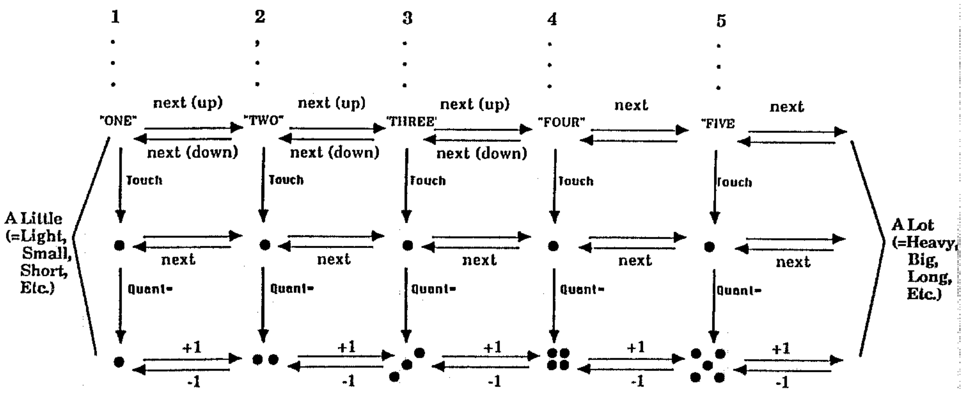

FIGURE 3 Cognitive structure underlying 6-year-old's numerical understanding. Dotted lines indicate optional (i.e., nonuniversal) notational knowledge.

"scales," and by the age of 10, they are capable of developing an explicit rule for relating one such line or scale to another.

A similar transition is presumed to occur during the same time period in the social domain. Children between 3½ and 5 years old have one structure for representing the simple social scripts with which they are familiar and encoding the events in these scripts in linguistic narrative (Nelson & Gruendel, 1981). They have a second structure for diagnosing what is going on in someone else's mind (Astington, Olson, & Harris, 1989). Children between 5 and 7 years old, however, begin to integrate these two structures. One result of this integration is that they can generate stories that involve a simple story line (McKeogh, 1992b). Another result is that they can offer an explanation or prediction about an event taking place in a social script by reference to a mental state rather than just the event that precedes or follows it (Goldberg-Reitman, 1992). Finally, as children's mental story line becomes more automated, they become capable of focusing on two such story lines (around 8 years) and then developing an explicit rule for relating such story lines to each other.

COMPARISON BETWEEN THE CONTEMPORARY NOTION OF A CENTRAL CONCEPTUAL STRUCTURE AND THE CLASSICAL NOTION OF A "STRUCTURE OF THE WHOLE"

Like Piaget's *structures of the whole,* the central conceptual structures that have been hypothesized are internalized sets of operations that are arranged into coherent systems, which change only gradually in their constituent make-up. They also have different characteristic forms at different developmental stages and are used to make sense of or learn new things about the external world. The process of structural formation that has been proposed is similar, too. Each higher order structure is assembled out of lower order structures, which become differentiated and coordinated via a process that includes an autoregulative component (e.g., equilibration and reflexive abstraction) and is activated by the universal human experience of trying to make sense of, or abstract invariance from, the normal flux of human life.

Although there are many similarities between the two constructs, there are a number of important differences as well. The structures now being proposed are organized sets of concepts and conceptual relations, not logical ones. Also, although they are universal with regard to sequence, they are potentially specific with regard to their form and incidence of occurrence. Finally, they are acquired via a process that invariably includes some form of social interaction, as well as auto-regulative activity. When cognitive structure and structural transformations are reconceptualized in this fashion, most of the theoretical problems that were associated with Piaget's theory either disappear or are radically attenuated (Case, 1992a, chap. 19).

EDUCATIONAL APPLICATIONS OF THE RESTRUCTURED THEORY

To date, four educational applications of the restructured developmental theory have been suggested.

Assessment

The first application is in the area of assessment. If central conceptual structures really do exist and develop in the fashion that has been hypothesized, it should be possible to create new assessment devices: ones in which the knowledge that is assessed is coherent and conforms to the developmental properties that have been outlined. Such tests would require that at least some of the questions children are asked be novel, to ensure that what is being assessed is a set of relations that the children genuinely understand. The new tests would also require that the questions be theoretically based and designed to determine children's overall sense of the domain they are designed to evaluate.

In a recent study, Okamoto (1992) created such a measure of 4- to 10-year-olds' numerical knowledge. At the unidimensional level, she asked questions such as, "Which is more, 5 or 4?" or "What number comes after 5, when you're counting backwards?" At the bidimensional level, she asked questions such as, "What is the largest two-digit number you can think of?" And at the integrated bidimensional level, she asked questions such as, "Which difference is bigger, the difference between 7 and 9 or the difference between 2 and 5?" None of the questions were very difficult in a computational sense, but all required a genuine understanding of the principles underlying one of the developmental structures that have been hypothesized.

What Okamoto's (1992) data showed were that (a) the items on her new measure formed an almost perfect Guttman scale (Guttman, 1945), and (b) the items at each level also met Goodman's (1975) test for the presence of a distinctive latent structure. When the test was used to assess the results of different school curricula, it also showed an interesting pattern of results. For example, when two middle-class populations were compared—one in Tokyo and one in California—they were found to be very different in terms of their computational facility, as measured by standard assessment devices (e.g., Stevenson & Stigler, 1992), but to be equivalent in terms of their general level of numerical development, as assessed by Okamoto's new measure. By contrast, when populations of different social backgrounds were assessed within the U.S., they showed large and significant differences on both measures. Such findings suggest that the structures we have isolated are less dependent on the particular curriculum that is used than traditional achievement measures, but that they are strongly dependent on the general nature of children's quantitative and symbolic experience. The results also have interesting implications for the attempt that is currently underway in the U.S. to reform the teaching of mathematics and to bring it to a level achieved by other countries.

Early Childhood Education

A second educational application of the restructured theory has to do with preschool education. As might be expected, structures, such as the mental number line (Figure 3), are crucial, if children are to profit from their first exposure to formal arithmetic. Unfortunately, however, many children enter first grade without already having these structures in place, particularly in populations that are at risk for school failure. In a recent study, Griffin (1992) showed that children can be helped to construct such a structure by means of a series of 30 game-like exercises, which combine the use of varied representational devices (e.g., thermometers, number cards, and dice) with goals that elicit a high degree of affective involvement. When exposed to this sort of curriculum, children at risk for school failure have been found to do better in first grade mathematics than matched controls and to be ranked by their teachers as having better number sense, up to 1 full year later.

Creation of New Curricula

A third use to which the restructured theory can be put is to sketch out the general sequence of attainments that should be expected in a new curriculum domain. Case and Sowder (1990) used the theory this way to map out the general sequence of attainments that could be expected with regard to computational estimation. Krohn and Bushey (1992) sketched out a similar sequence for children's early fractional knowledge. Finally, McKeough (1992a) suggested a general set of characteristics to develop new curricula that would alter both the content that is covered and the techniques that are used for doing so. One of the most prominent of her suggestions was that conceptually oriented exercises and drill-and-practice exercises should not be seen as opposed but as complementary.

Remedial Instruction and Tutoring

No matter how effective a curriculum, there will always be students who, due to illness or some other problem, fail to master the material that is presented when this is expected of them. A final application of the restructured theory is to capture the conceptual "essentials" of this material and to help children overcome their particular disadvantage. Such an approach has been described in some detail by McKeough (1992a) for the field of early language arts. And the principles it entails have been applied to the domain of mathematics in a number of different clinical studies (e.g., Groo, 1992; Krohn, 1990; Peternick, 1992; Suyemoto, 1992).

CONTEMPORARY THEORIES OF LEARNING AND DEVELOPMENT

At the beginning of this article, I suggested that Piaget's classical theory lost much of its power to serve as the basis for distinctive educational reform, as learning theory caught up with and surpassed Piagetian theory in its emphasis on cognition and in its technical sophistication. The question to which I would like to return is whether the sorts of educational applications mentioned in the previous section are distinctive or could be derived with equal ease from contemporary learning theory.

It seems to me that all the applications could be explained in the context of contemporary learning theory by interpreting a central conceptual structure as a powerful organizing schema, which combines aspects of procedural and conceptual knowledge and serves to channel children's epistemic activity. On the other hand, I think it unlikely that the structures would have been discovered in the context of contemporary learning theory—even if formulated in this fashion—for the simple reason that contemporary learning theory is built on the same general assumptions about development as was its predecessors; namely, development can be reduced to cumulative learning, and learning can be attributed, in its entirety, to the specific experiences that children encounter in specific contexts. By contrast, the construct of a central conceptual structure was formulated within the neo-Piagetian tradition, where children's cognitive systems are not presumed to result in any straightforward or simple fashion from their specific experience; rather, they result from the child's active reworking of this experience under the constraints imposed by their general level of development on the one hand and their existing specific structures on the other.

Because the formalism that has been used to describe children's central conceptual structures has been drawn from contemporary schema theory, and because the importance of learning in structural acquisition has been explicitly acknowledged, children's central conceptual structures can, indeed, be thought of as similar to the schemas that have been studied in contemporary learning theory. However, due to the differences just mentioned, they remain distinctive in the following three respects. First, the content and organization of the structures is not completely determined by the experience to which children are exposed: That is to say, it is not a simple reflection or "copy" of this experience. Second, the structures apply across a much broader range of content than schemas that represent particular problem types, content domains, or even academic disciplines. Third, the acquisition of these structures is tied to children's general level of development, not just to their formal schooling or other specific experience. Given this pattern of similarities and differences, it seems reasonable to suggest that analysis of children's domain-specific schemas should proceed hand in hand with analysis of their more general conceptual structures and that both sorts of analysis should be seen as enriching our understanding of the educational process.

REFERENCES

Astington, J. W., Olson, D., & Harris, P. (Eds.). (1989). *The child's theory of mind.* New York: Cambridge University Press.

Bereiter, C., & Scardamalia, M. (1985). Cognitive coping strategies and the problem of inert knowledge. In S. Chipman, J. W. Segal, & R. Glaser (Eds.), *Thinking and learning skills* (Vol. 2, pp. 65–81). Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.

Capodilupo, A. M. (1992). A neo-structural analysis of children's response to instruction in the sight-reading of musical notation. In R. Case (Ed.), *The mind's staircase: Exploring the conceptual underpinnings of children's thought and knowledge* (pp. 66–116). Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.

Carey, S., & Smith, C. (1993/this issue). On understanding the nature of scientific knowledge. *Educational Psychologist, 28,* 235–251.

Case, R. (1974). Structures and strictures: Some functional limitations on the course of cognitive growth. *Cognitive Psychology, 6,* 544–573.

Case, R. (1985). *Intellectual development: Birth to adulthood.* New York: Academic.

Case, R. (1992a). *The mind's staircase: Exploring the conceptual underpinnings of children's thought and knowledge.* Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.

Case, R. (1992b). The role of the frontal lobes in the regulation of cognitive development. *Brain and Cognition, 20,* 51–73.

Case, R., & Bereiter, C. (1984). From behaviorism to cognitive behaviorism to cognitive development: Steps in the evolution of instructional design. *Instructional Science, 13,* 141–158.

Case, R., & Griffin, S. (1990). Child cognitive development: The role of central conceptual structures in the development of scientific and social thought. In C. A. Hauert (Ed.), *Advances in psychology: Developmental psychology* (pp. 193–230). Amsterdam: Elsevier.

Case, R., & McKeough, A. (1990). Schooling and the development of central conceptual structures. *International Journal of Education, 13,* 835–855.

Case, R., & Sandieson, R. (1988). A developmental approach to the identification and teaching of central conceptual structures in the middle grades. In J. Hiebert & M. Behr (Eds.), *Research agenda in mathematics education: Number concepts and operations in the middle grades* (pp. 236–270). Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.

Case, R., Sandieson, R., & Dennis, S. (1986). Two cognitive developmental approaches to the design of remedial instruction. *Cognitive Development, 1,* 293–333.

Case, R., & Sowder, J. (1990). The development of computational estimation: A neo-Piagetian analysis. *Cognition and Instruction, 7,* 79–104.

Chi, M. T. H., & Rees, E. (1983). A learning framework for development. *Contributions to Human Development, 9,* 71–107.

Collis, K., & Biggs, J. (1982). *The SOLO taxonomy.* New York: Academic.

Damon, W. (1973). *The child's conception of justice as related to logical thought.* Unpublished doctoral dissertation, University of California, Berkeley.

Davis, R., Noddings, N., & Maher, C. (1990). Constructivist views on the teaching and learning of mathematics. *Monographs of the Journal for Research in Mathematics Education, 4.*

Fischer, K. W. (1980). A theory of cognitive development: The control and construction of hierarchies of skills. *Psychological Review, 87,* 477–531.

Gagné, R. M., & Briggs, L. J. (1979). *Principles of instructional design* (2nd ed.). New York: Holt.

Gelman, R. (1978). Counting in the preschooler: What does and what does not develop? In R. Siegler (Ed.), *Children's thinking: What develops?* (pp. 213–242). Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.

Ginsburg, H., & Opper, S. (1969). *Piaget's theory.* New York: Prentice-Hall.

Goldberg-Reitman, J. (1992). Young girls' conceptions of their mothers' roles. In R. Case (Ed.), *The mind's staircase: Exploring the conceptual underpinnings of children's thought and knowledge* (pp. 135–153). Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.

Goodman, L. A. (1975). A new model for scaling response patterns: An application of the quasi-independence concept. *Journal of the American Statistical Association, 70,* 755–768.

Griffin, S. A. (1992, April). *Rightstart: Providing the central conceptual prerequisites for first formal learning of arithmetic to students at risk for school failure.* Paper presented at the annual meeting of the American Educational Research Association, San Francisco.

Griffin, S. A., Case, R., & Sandieson, R. (1992). Synchrony and asynchrony in the acquisition of children's everyday mathematical knowledge. In R. Case (Ed.), *The mind's staircase:*

Exploring the conceptual underpinnings of children's thought and knowledge (pp. 75–97). Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.

Groo, M. (1992). *Constructing a number line.* Unpublished manuscript, Stanford University.

Guttman, L. (1945). The basis for scalogram analysis. In S. A. Stouffer, L. Guttman, E. A. Suchman, P. F. Lazarsfeld, S. A. Star, & J. A. Clausen (Eds.), *Measurement and prediction.* New York: Wiley.

Halford, G. S. (1982). *The development of thought.* Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.

Kamii, C. K., & Radin, N. L. (1967). A framework for a preschool curriculum based upon Piaget's theory. *Journal of Creative Behavior, 1,* 314–324.

Karplus, R. (1964). The science curriculum improvement study. In R. E. Ripple & V. N. Rockcastle (Eds.), *Piaget rediscovered* (pp. 113–118). Ithaca, NY: Cornell School of Education Press.

Krohn, C. (1990). An adolescent's understanding of decimal number: Clinical interview and intervention. In R. Case (Ed.), *The role of central conceptual structures in the development of children's numerical, literary, and spatial thought* (pp. 164–193, Report). Chicago: Spencer Foundation.

Krohn, C., & Bushey, B. (1992, April). *The acquisition of fractional number knowledge: A developmental analysis.* Paper presented at the annual meeting of the American Educational Research Association, San Francisco.

Liu, P. A. (1981). *An investigation of the relationship between qualitative and quantitative advances in the cognitive development of preschool children.* Unpublished doctoral dissertation, Ontario Institute for Studies Education.

Marini, Z. (1992). Synchrony and asynchrony in the development of children's scientific reasoning. In R. Case (Ed.), *The mind's staircase: Exploring the conceptual underpinnings of children's thought and knowledge* (pp. 55–73). Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.

McKeough, A. (1992a, April). *Implications of the notion of a central conceptual structure for instructional planning.* Paper presented at the annual meeting of the American Educational Research Association, San Francisco.

McKeough, A. (1992b). A neo-structural analysis of children's narrative and its development. In R. Case (Ed.), *The mind's staircase: Exploring the conceptual underpinnings of children's thought and knowledge* (pp. 171–189). Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.

Minstrell, J. (1990, April). *A teaching system for diagnosing student' conceptions and prescribing relevant instruction.* Paper presented at the annual meeting of the American Educational Research Association, Boston.

Nelson, K., & Gruendel, J. (1981). Generalized event representations: Basic building blocks of cognitive development. In M. E. Lamb & A. L. Brown (Eds.), *Advances in developmental psychology* (Vol. 1, pp. 131–158). Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.

Okamoto, Y. (1992, April). *Implications of the notion of central conceptual structures for assessment.* Paper presented at the annual meeting of the American Educational Research Association, San Francisco.

Pascual-Leone, J. (1969). *Cognitive development and cognitive style.* Unpublished doctoral dissertation, University of Geneva.

Pascual-Leone, J. (1970). A mathematical model for the transition rule in Piaget's development stages. *Acta Psychologica, 32,* 301–345.

Pascual-Leone, J. (1988). Organismic processes for neo-Piagetian theories: A dialectical causal account of cognitive development. In A. Demetriou (Ed.), *The neo-Piagetian theories of cognitive development: Toward an integration* (pp. 25–65). Amsterdam: North Holland.

Peternick, L. (1992). *Valuing numbers.* Unpublished manuscript, Stanford University.

Piaget, J. (1970). Piaget's theory. In P. H. Mussen (Ed.), *Carmichael's handbook of child development* (pp. 703–732). New York: Wiley.

Resnick, L. (1983). A developmental theory of number understanding. In H. P. Ginsburg (Ed.), *The development of mathematical thinking* (pp. 110–153). New York: Academic.

Scardamalia, M., & Bereiter, C. (1988). Fostering the development of self-regulation in children's knowledge processing. In S. Chipman, J. W. Segal, & R. Glaser (Eds.), *Thinking and learning skills* (pp. 563–578). Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.

Siegler, R. S. (1976). Three aspects of cognitive development. *Cognitive Psychology, 8,* 481–520.

Simon, D. P., & Simon, H. A. (1978). Individual differences in solving physics problems. In R. S. Siegler (Ed.), *Children's thinking: What develops?* (pp. 324–348). Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.

Stevenson, H., & Stigler, J. (1992). *The learning gap.* New York: Summit Books.

Strauss, S. (1993/this issue). Teachers' pedagogical content knowledge about children's minds and learning: Implications for teacher education. *Educational Psychologist, 28,* 279–290.

Suyemoto, L. (1992). *Counting money.* Unpublished manuscript, Stanford University.

Wainryb, C., & Turiel, E. (1993/this issue). Conceptual and informational features in moral decision making. *Educational Psychologist, 28,* 205–218.